



Simultaneous equations in three unknowns

About 90 minutes No calculator 76 marks

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Solve the simultaneous equations $5x + 2y = 19$ and $3x - 2y = 5$.

A2 Needed today [2 marks]

$x + 2y - z = 4$ and $z = 3x - y$.
Substitute for z and simplify, to give an equation in x and y only.

Answer $x = \dots\dots$ $y = \dots\dots$

Answer $\dots\dots\dots$

- A3** Last lesson [2 marks]

Work out the limiting value, as $n \rightarrow \infty$, of the sequence with n th term $\frac{12n - 5}{3n + 4}$.

A4 From chapter 3 [2 marks]

$f(x) = \frac{2x + 5}{3}$. Work out $f^{-1}(x)$.

Answer $f^{-1}(x) = \dots\dots\dots$

Answer $\dots\dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve the simultaneous equations
 $2x + 3y + z = 4$, $x - 2y + 3z = 13$ and
 $3x + y - 2z = -1$.

Hint Number the equations. Pick one unknown (here z) and remove it from two different pairs of equations. That leaves two equations in x and y .

- B1** Solve the simultaneous equations $3x + 2y - z = 3$,
 $2x - y + 3z = 0$ and
 $x + 3y + 2z = 15$. [5 marks]

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

EXAMPLE 2

Solve the simultaneous equations

$$2a + 3c = 13, a - b + 2c = 9 \text{ and}$$

$$3a + 2b - c = 10.$$

Hint The first equation has no b in it already. So eliminate b from the other two, and you have two equations in a and c .

B2 Solve the simultaneous [4 marks]

$$\text{equations } 4p - r = 16,$$

$$2p + 3q + r = 5 \text{ and}$$

$$p - q + 2r = -6.$$

Answer $p = \dots\dots$ $q = \dots\dots$ $r = \dots\dots$

EXAMPLE 3

Solve the simultaneous equations

$$z = 2x - y + 1, 3x + 2y = z + 8 \text{ and}$$

$$x + y + 2z = 15.$$

Hint The first equation already gives z . Substitute it into the other two equations, then tidy each into the form $ax + by = c$.

B3 Solve the simultaneous [4 marks]

$$\text{equations } y = 2z - x + 1,$$

$$2x + 3z = y - 6 \text{ and}$$

$$4x - y + z = -12.$$

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

EXAMPLE 4

$x = 2, y = -1, z = 3$ is the solution of the simultaneous equations $ax + by + cz = 14$, $bx + cy + az = 5$ and $cx + ay - bz = 7$. Work out the values of a, b and c .

Hint Put the values of x, y and z in. Now a, b and c are the unknowns, in three ordinary equations.

B4 $x = 3, y = -2, z = 1$ is the [5 marks]

solution of the simultaneous

$$\text{equations } ax + by + cz = -5,$$

$$ax + bz - cy = 9 \text{ and}$$

$$bx + ay - cz = 12. \text{ Work out the values of } a, b \text{ and } c.$$

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

EXAMPLE 5

The curve $y = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 8)$ and $(2, 5)$. Work out the values of a , b and c .

Hint Each point gives one equation: put its x and y into $y = ax^2 + bx + c$.

B5 The curve $y = ax^2 + bx + c$ [4 marks]
passes through the points $(1, -4)$, $(-2, 17)$ and $(3, 2)$.
Work out the values of a , b and c .

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

c Practice

Level 2

C1 Solve the simultaneous equations $x + 2y - z = -1$, $3x - y + 2z = 13$ and $2x + 3y + z = -2$ by first eliminating y . [5 marks]

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

C2 At a theatre, 2 adult tickets, 3 child tickets and 1 senior ticket cost £54. 1 adult ticket, 2 child tickets and 2 senior tickets cost £44. 3 adult tickets, 1 child ticket and 1 senior ticket cost £52. Work out the cost of each type of ticket. [5 marks]

Answer adult £ ... child £ ... senior £ ...

C3 Solve the simultaneous equations $4x + y + z = 3$, $2x - 2y + 3z = -11$ and $6x + y - 2z = 10$. [5 marks]

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

C4 The 2nd, 5th and 6th terms of a quadratic sequence are 6, 51 and 74. Work out the n th term of the sequence. [4 marks]

Answer

D Exam-style questions

AQA exam style

- D1** Solve the simultaneous equations $4a + b - 3c = -4$, $2a + 3b + c = -7$ and $2a - 2b + 5c = 21$. Do not use trial and improvement. You must show your working. [5 marks]

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

- D2** Sam says, " $x = 2$, $y = 1$, $z = 3$ is the solution of these equations, because it works in the first two."

$$x + y + z = 6, \quad 2x + y = 5, \quad 3x - y + 2z = 4$$

- (a) Show that Sam is wrong. [1 mark]

- (b) Solve the simultaneous equations. You must show your working. [4 marks]

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

Total for D2: 5 marks

- D3** The curve $y = ax^2 + bx + c$ passes through the points $(-1, 10)$, $(2, 1)$ and $(3, 6)$.

- (a) Work out the values of a , b and c . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

- (b) Hence show that the line $y = 3x - 5$ is a tangent to the curve. [3 marks]

Total for D3: 7 marks

D4 $a + b + c = 7$, $a + 2b + 4c = 21$, $a + 3b + 9c = 43$

(a) Solve the simultaneous equations.

[4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

(b) Hence write down an expression for the n th term of the quadratic sequence
7, 21, 43, ...

[1 mark]

Answer $\dots\dots\dots$

Total for D4: 5 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A café sells tea at £2, coffee at £3 and cake at £5. A group buys 20 items for £61 [5 marks]
in total. They buy at least one of each, and more coffees than teas. Work out
every possible number of teas, coffees and cakes.

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call the numbers t , c and k . You have only two equations, so look for whole-number answers.
- Hint 2** $t + c + k = 20$ and $2t + 3c + 5k = 61$. Take $2 \times$ the first from the second: $c + 3k = 21$.
- Hint 3** So $c = 21 - 3k$ and $t = 20 - c - k = 2k - 1$. Both at least 1 means $1 \leq k \leq 6$; $c > t$ means $21 - 3k > 2k - 1$, so $k \leq 4$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can solve three simultaneous equations by eliminating one unknown. ☐ ☐ ☐
- I can rearrange and substitute when an equation is not in the usual form. ☐ ☐ ☐
- I can find unknown coefficients, a quadratic curve or an n th term from three facts. ☐ ☐ ☐
- I can form and solve three equations from a problem in context. ☐ ☐ ☐