

Simultaneous equations in two unknowns

About 80 minutes No calculator 75 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-simultaneous-equations-in-two-unknowns/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks]

Expand and simplify
 $(2x - 5)^2 - x(x - 4)$.

A2 GCSE Higher [2 marks]

Make y the subject of
 $3x - 4y = 10$.

Answer _____

Answer $y = \dots\dots$

- A3** Last lesson [2 marks]

Use the discriminant to show that $3x^2 - 5x + 4 = 0$ has no real roots.

A4 From chapter 3 [2 marks]

$f(x) = 2x^2 - 3$. Solve $f(x) = 47$.

Answer $x = \dots\dots$ or $x = \dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve the simultaneous equations

$$3x + 4y = 13 \quad \text{and} \quad 5x - 6y = 47.$$

Hint Make the y terms match: multiply the first by 3 and the second by 2. The signs differ, so add.

B1 Solve the simultaneous [3 marks]

equations $5x + 3y = 5$ and
 $2x - 7y = -39$.

Answer $x = \dots\dots$ $y = \dots\dots$

EXAMPLE 2

Solve the simultaneous equations

$$y = x^2 - 2x - 3 \quad \text{and} \quad y = 2x + 9.$$

Hint Both expressions equal y , so set them equal. Then use the LINEAR equation to find each y .

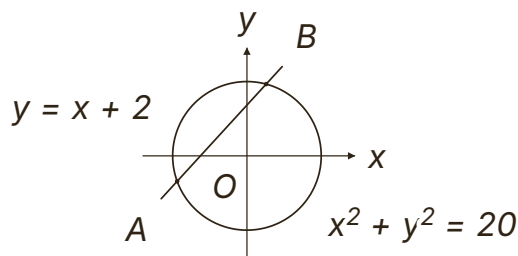
B2 Solve the simultaneous [4 marks]

equations $y = 2x^2 + x - 4$
and $y = 3x + 8$.

Answer $(\dots, \dots)$ and $(\dots, \dots)$

EXAMPLE 3

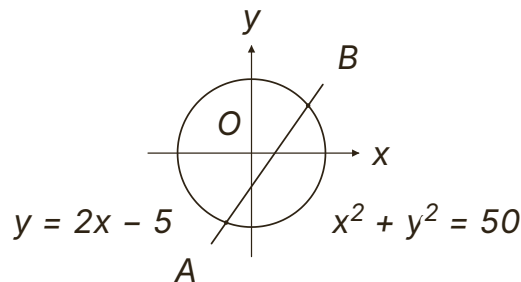
The line $y = x + 2$ meets the circle $x^2 + y^2 = 20$ at the points A and B . Work out the coordinates of A and B .



Not drawn accurately

Hint Put $x + 2$ in place of y in the circle's equation. Expand the bracket before you collect terms.

B3 The line $y = 2x - 5$ meets the circle $x^2 + y^2 = 50$ at the points A and B . Work out the coordinates of A and B . [4 marks]



Not drawn accurately

Answer (.....,) and (.....,)

EXAMPLE 4

Solve the simultaneous equations

$$x^2 - xy + y^2 = 7 \quad \text{and} \quad x + y = 1.$$

Hint Write $y = 1 - x$ and substitute. Every term must be expanded with care: $-x(1 - x) = -x + x^2$.

B4 Solve the simultaneous equations [5 marks]

$$2y^2 - xy - 2x^2 = 13 \quad \text{and} \\ y - x = 2.$$

Answer (... , ...) and (... , ...)

EXAMPLE 5

The line $y = x + k$ is a tangent to the circle $x^2 + y^2 = 18$. Work out the two possible values of k .

Hint A tangent meets the circle at exactly one point, so the quadratic in x has a repeated root: $b^2 - 4ac = 0$.

B5 The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 20$. Work out the two possible values of k . [4 marks]

Answer $k = \dots\dots$ or $k = \dots\dots$

- C1** Four coffees and three muffins cost £13.90. Two coffees and five muffins cost £12.20. Work out the cost of three coffees and two muffins. [4 marks]

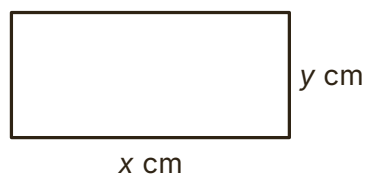
Answer £

- C2** The simultaneous equations $ax + by = 11$ and $bx - ay = 7$ have the solution $x = 3, y = -1$. Work out the values of a and b . [3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

- C3** Show that the line $y = x - 6$ does not meet the curve $y = x^2 - 3x$. [2 marks]

- C4** A rectangle has a perimeter of 38 cm and an area of 84 cm^2 . Its length is x cm and its width is y cm, where $x > y$. Form a pair of simultaneous equations and solve them to work out x and y . [4 marks]



Not drawn accurately

- C5** Solve the simultaneous equations $\frac{x}{4} + \frac{y}{3} = 4$ and $\frac{x}{2} - \frac{y}{6} = 3$. [3 marks]

Answer $x = \dots\dots$ $y = \dots\dots$

- C6** Solve the simultaneous equations $xy = 18$ and $y = 2x - 5$. [4 marks]

Answer $(\dots, \dots)$ and $(\dots, \dots)$

D1 Solve the simultaneous equations

[5 marks]

$$2x^2 - xy + 2y^2 = 30 \quad x - y = 3$$

Do not use trial and improvement. You must show your working.

Answer $x = \dots\dots$, $y = \dots\dots$ and $x = \dots\dots$, $y = \dots\dots$

D2 Sam says, "The line $y = 4x - 7$ crosses the curve $y = x^2 - 2x + 2$ at two points, because a straight line always meets a *U*-shaped curve twice."

[3 marks]

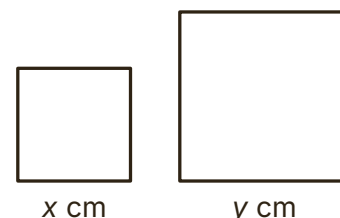
Show that Sam is wrong, and work out the coordinates of every point where the line meets the curve.

Answer $(\dots\dots, \dots\dots)$

D3 Two square tiles have sides x cm and y cm, where $x < y$. The total of their perimeters is 92 cm. The total of their areas is 277 cm^2 .

(a) Show that $x^2 - 23x + 126 = 0$.

[3 marks]



(b) Hence work out the side of each tile.

[2 marks]

Not drawn accurately

Answer $x = \dots\dots$ $y = \dots\dots$

Total for D3: 5 marks

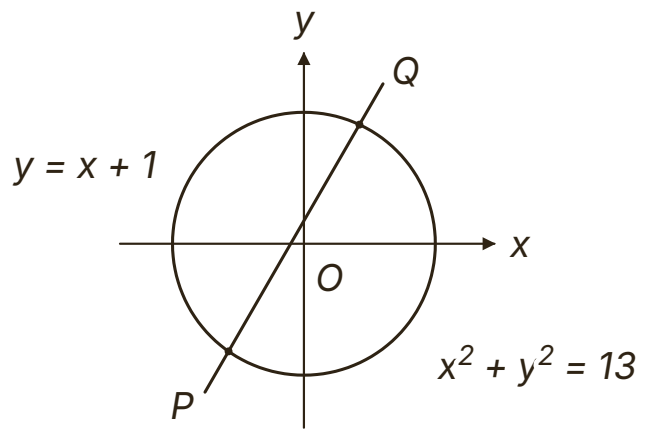
D4 How many points of intersection do the graphs of $y = x^2 + 2$ and $y = 2x$ have?

[1 mark]

Circle your answer. 0 1 2 3

D5 The line $y = x + 1$ meets the circle $x^2 + y^2 = 13$ at the points P and Q .

- (a) Work out the coordinates of P and Q . [4 marks]



Answer (.....,) and (.....,)

- (b) Work out the length of PQ . Give your answer in the form $a\sqrt{2}$. [2 marks]

Not drawn accurately

Answer $\sqrt{2}$

Total for D5: 6 marks

D6 The graphs of $y = x^2 + px + q$ and $y = 2x + 1$ meet at the points $(1, 3)$ and $(4, 9)$. [3 marks]
Work out the values of p and q .

Answer $p = \dots\dots$ $q = \dots\dots$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve the simultaneous equations $x + y = 5$ and $x^3 + y^3 = 35$. [4 marks]

Answer $x = \dots\dots$, $y = \dots\dots$ and $x = \dots\dots$, $y = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find the value of xy first. Think about how $(x + y)^3$ expands.
Hint 2 $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$, so $125 = 35 + 3xy \times 5$.
Hint 3 $15xy = 90$ gives $xy = 6$. Then x and y are the roots of $t^2 - 5t + 6 = 0$, which are 2 and 3.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve two linear equations. ☐ ☐ ☐

I can solve a linear and a quadratic equation as pairs. ☐ ☐ ☐

I can tell if a line meets, touches or misses a curve. ☐ ☐ ☐

I can form simultaneous equations from a problem. ☐ ☐ ☐