

Solving trigonometric equations

About 80 minutes Calculator allowed 71 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-solving-trigonometric-equations/

Date _____

A Do Now

Four questions to start. Two of them use what you need today.

- A1** Needed today [2 marks]

Solve $6y^2 + y - 2 = 0$.

A2 Last lesson [2 marks]

You are given that $\cos 25^\circ = c$.
Write down two angles x , with
 $0^\circ \leq x \leq 360^\circ$, for which
 $\cos x = -c$.

Answer $y = \dots\dots$ or $y = \dots\dots$

Answer $x = \dots\dots$ and $x = \dots\dots$

- A3** From chapter 4 [2 marks]

Solve $x^2 - 3x - 5 = 0$. Give your answers to 2 decimal places.

A4 From chapter 6 [2 marks]

Triangle ABC has a right angle at B . $AB = 7$ cm and $BC = 4$ cm. Work out angle BAC . Give your answer to 1 decimal place.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

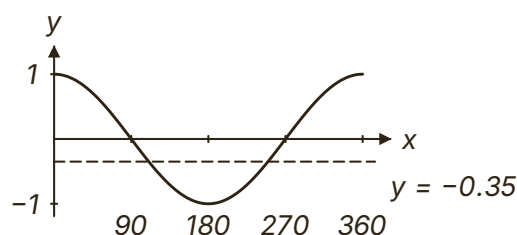
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Give answers to 1 decimal place where they are not exact.

EXAMPLE 1

Solve $\cos x = -0.35$ for $0^\circ \leq x \leq 360^\circ$.



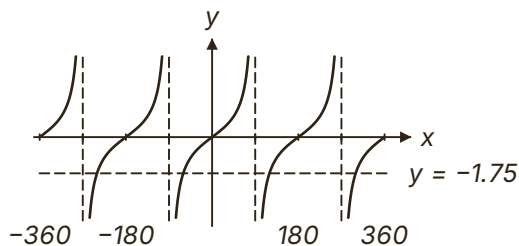
Hint The calculator gives the principal value. The cosine graph is symmetric about $x = 180^\circ$, so the second angle is 360° minus the first.

- B1** Solve $\sin x = -0.45$ for $0^\circ \leq x \leq 360^\circ$. [2 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

EXAMPLE 2

Solve $4 \tan x + 7 = 0$ for $-360^\circ \leq x \leq 360^\circ$.



Hint Make $\tan x$ the subject first. The tangent repeats every 180° : add and take away 180° until you leave the interval.

B2 Solve $9 \sin x + 4 = 0$ for $0^\circ \leq x \leq 720^\circ$. [3 marks]

Answer $x = \dots\dots$

EXAMPLE 3

Solve $5 \tan^2 x = 2$ for $0^\circ \leq x \leq 360^\circ$.

Hint $\tan^2 x = 0.4$, so $\tan x = \sqrt{0.4}$ or $\tan x = -\sqrt{0.4}$. Solve each one.

B3 Solve $9 \sin^2 x = 5$ for $0^\circ \leq x \leq 360^\circ$. [3 marks]

Answer $x = \dots\dots$

EXAMPLE 4

Solve $2 \cos^2 x - 3 \cos x - 2 = 0$ for $0^\circ \leq x \leq 360^\circ$.

Hint Write $c = \cos x$ and factorise $2c^2 - 3c - 2$. A value of $\cos x$ outside -1 to 1 gives no solutions.

B4 Solve $6 \sin^2 x + \sin x - 2 = 0$ for $0^\circ \leq x \leq 360^\circ$. [4 marks]

Answer $x = \dots\dots$

EXAMPLE 5

$f(t) = 2t^3 - 3t^2 - 3t + 2$. Show that $f(2) = 0$.

Hence solve

$$2\sin^3 x - 3\sin^2 x - 3\sin x + 2 = 0 \text{ for}$$

$$0^\circ \leq x \leq 360^\circ.$$

Hint $(t - 2)$ is a factor. Divide to find the quadratic factor, factorise it, then replace t by $\sin x$.

B5 $g(t) = 3t^3 - 7t^2 - 7t + 3$. Show [5 marks]
that $g(3) = 0$. Hence solve
 $3\cos^3 x - 7\cos^2 x - 7\cos x + 3 = 0$
for $-180^\circ \leq x \leq 180^\circ$.

Answer $x = \dots\dots$

c Practice

Level 2

C1 Solve $(2\sin x - 1)(\cos x + 1) = 0$ [3 marks]
for $0^\circ \leq x \leq 360^\circ$.

Answer $x = \dots\dots$

C2 Solve $2\tan^2 x = 5\tan x$ for [4 marks]
 $0^\circ \leq x \leq 360^\circ$.

Answer $x = \dots\dots$

C3 Do not use a calculator. Solve [2 marks]
 $2\cos x + \sqrt{3} = 0$ for
 $0^\circ \leq x \leq 360^\circ$.

Answer $x = \dots\dots$ or $x = \dots\dots$

C4 Do not use a calculator. Work [2 marks]
out the value of x , where
 $0^\circ \leq x \leq 90^\circ$, for which
 $4\sin^2 x = 3$.

Answer $x = \dots\dots$

C5 Without solving them, write [2 marks]
down how many solutions each
equation has for $0^\circ \leq x \leq 1080^\circ$.
Give a reason for each.

(i) $5\cos x = 2$

(ii) $3\sin x + 4 = 0$

Answer (i) ... (ii) ...

C6 Solve $7\cos x + 2 = 0$ for [3 marks]
 $-360^\circ \leq x \leq 0^\circ$.

Answer $x = \dots\dots$ or $x = \dots\dots$

- D1** Mia solves $3 \sin^2 x = \sin x$ for $0^\circ \leq x \leq 360^\circ$. She divides both sides by $\sin x$ to get $\sin x = \frac{1}{3}$, and gives the answers $x = 19.5^\circ$ and $x = 160.5^\circ$. [3 marks]

Explain why Mia's answer is not complete. Write down the other solutions.

Answer other solutions $x = \dots\dots$

- D2** One solution of $\tan 2x = 1$ is $x = 22.5^\circ$. Circle another solution. [1 mark]
- 45° 67.5° 112.5° 157.5°

Answer

- D3** Solve $3 \cos^2 x + 5 \cos x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place. You must show your working. [5 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

- D4** Answer both parts.

- (a) Factorise $4y^2 - 9y + 2$. [2 marks]

Answer

- (b) Hence solve $4 \tan^2 x - 9 \tan x + 2 = 0$ for $-180^\circ \leq x \leq 180^\circ$. Give your answers to 1 decimal place. [4 marks]

Answer $x = \dots\dots$

Total for D4: 6 marks

D5 A seat on a fairground wheel is h metres above the ground, where

$$h = 12 - 10 \cos x$$

and x° is the angle the wheel has turned through since the seat was at the bottom.

- (a) Write down the greatest height of the seat. [1 mark]

Answer m

- (b) Work out the values of x , for $0 \leq x \leq 360$, for which the seat is 18 m above the ground. Give your answers to 1 decimal place. [3 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

- (c) The wheel turns through two full turns, so $0 \leq x \leq 720$. Work out the largest value of x for which the seat is 18 m above the ground. [1 mark]

Answer $x = \dots\dots$

Total for D5: 5 marks

- D6** Jo says, "The equation $4 \sin x + 7 = 2$ has two solutions for $0^\circ \leq x \leq 360^\circ$." [2 marks]

Is Jo correct? Give a reason for your answer.

Answer

- D7** Do not use a calculator. Solve $2 \sin^2 x - \sin x - 1 = 0$ for $360^\circ \leq x \leq 720^\circ$. You must show your working. [4 marks]

Answer $x = \dots\dots$

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve $\cos^2(2x) = \frac{1}{4}$ for $0^\circ \leq x \leq 180^\circ$. Do not use a calculator.

[4 marks]

Answer $x = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

E1 Hint 1 Let $u = 2x$. If x goes from 0° to 180° , u goes from 0° to 360° .

Hint 2 $\cos u = \frac{1}{2}$ or $\cos u = -\frac{1}{2}$.

Hint 3 $u = 60, 300$ or $u = 120, 240$. Halve each one: $x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find all the solutions of $\sin x = k$, $\cos x = k$ or $\tan x = k$ in an interval. ☐ ☐ ☐

I can solve quadratic and cubic equations in $\sin x$, $\cos x$ or $\tan x$. ☐ ☐ ☐

I can tell when an equation has no solutions, and not lose solutions by dividing. ☐ ☐ ☐