



Stationary points

About 80 minutes No calculator 89 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Solve $3x^2 + 6x - 45 = 0$.

A2 Needed today [2 marks]

A rectangle has a perimeter of 40 cm and a width of x cm. Write an expression for its area in terms of x .

Answer

Answer

- A3** From chapter 8 [2 marks]

Work out the equation of the tangent to the curve $y = 4 + 3x - x^3$ at the point where $x = 2$.

A4 Last lesson [2 marks]

$y = x^3 - 5x^2 + 2x$. Work out the value of $\frac{d^2y}{dx^2}$ when $x = 3$.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the coordinates of the stationary point on the curve $y = 2x^2 - 12x + 7$. Is it a maximum or a minimum?

Hint A stationary point is where $\frac{dy}{dx} = 0$. Then look at the sign of $\frac{d^2y}{dx^2}$.

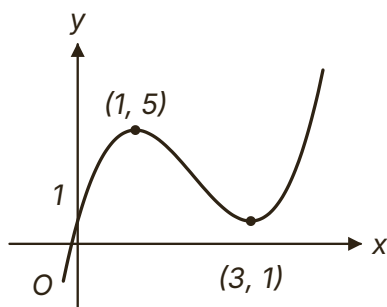
- B1** Work out the coordinates of the stationary point on the curve $y = 5 + 8x - 2x^2$. Is it a maximum or a minimum? [4 marks]

Answer (.....,.....) , a

EXAMPLE 2

Work out the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x + 1$.

Use $\frac{d^2y}{dx^2}$ to say whether each is a maximum or a minimum.



Not drawn accurately

Hint Set $\frac{dy}{dx} = 0$ and factorise. Put each x into y , then into $\frac{d^2y}{dx^2}$.

B2 [6 marks]

Work out the coordinates of the stationary points on the curve

$y = 2x^3 - 9x^2 - 24x + 10$. Use $\frac{d^2y}{dx^2}$ to say whether each is a maximum or a minimum.

EXAMPLE 3

$y = x^4 - 8x^2 + 3$. Work out the coordinates of the stationary points. Use the sign of the gradient on each side of each point to say whether it is a maximum or a minimum.

Hint Factorise $\frac{dy}{dx}$ fully. Then choose a test value of x between each pair of stationary points.

B3 [6 marks]

$y = 3x^4 - 4x^3 - 12x^2 + 1$.

Work out the coordinates of the three stationary points. Use the sign of the gradient on each side of each point to say whether it is a maximum or a minimum.

EXAMPLE 4

$y = 2x + \frac{18}{x}$ for $x > 0$. Work out the least value of y . Show that it is a minimum.

Hint Write $y = 2x + 18x^{-1}$. Use $x > 0$ when you solve $\frac{dy}{dx} = 0$.

B4 [5 marks]

$y = x^2 + \frac{128}{x}$ for $x > 0$. Work out the least value of y . Show that it is a minimum.

Answer

EXAMPLE 5

A farmer has 120 m of fence to make a rectangular pen against a long wall. The wall is one side, so the fence makes the other three sides. The two sides at right angles to the wall are each x m. Work out the greatest area the pen can have.

Hint Write the area A in terms of x only. Then solve $\frac{dA}{dx} = 0$ and check the sign of $\frac{d^2A}{dx^2}$.

- B5** An open box is made from a square sheet of card with sides 24 cm. A square of side x cm is cut from each corner and the sides are folded up. Work out the value of x that gives the greatest volume, and that volume. Show that it is a maximum. [6 marks]

Answer $x = \dots\dots$, volume $\dots\dots\dots \text{cm}^3$

c Practice

Level 2

- C1** The curve $y = ax^2 + bx + c$ [4 marks]
crosses the y -axis at $(0, 11)$ and has a minimum point at $(2, 3)$.
Work out the values of a , b and c .

- C2** The curve $y = px + qx^2$ passes [5 marks]
through the point $(2, 16)$. Its gradient at that point is 4. Work out the values of p and q . Hence work out the maximum value of y .

Answer $a = \dots\dots$, $b = \dots\dots$, $c = \dots\dots$

- C3** x and y are positive numbers [4 marks]
with $x + y = 18$. $P = x^2y$. Work out the greatest value of P .

- C4** The curve [4 marks]
 $y = x^3 + kx^2 + 24x - 5$ has a stationary point where $x = 2$. Work out the value of k . Hence work out the x -coordinate of the other stationary point.

Answer

Answer $k = \dots\dots$, $x = \dots\dots$

D1

Ella is finding the stationary points of $y = 4x^3 - 3x$. She writes:
 $\frac{dy}{dx} = 12x^2 - 3 = 0$, so $x = \pm\frac{1}{2}$. At $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = 24x = 12$, which is positive, so $(\frac{1}{2}, -1)$ is a maximum.

[3 marks]

Explain Ella's mistake. Work out the coordinates of the maximum point.

Answer (.....,.....)

D2 A curve has equation $y = x^3 - 6x^2 - 15x + 12$.

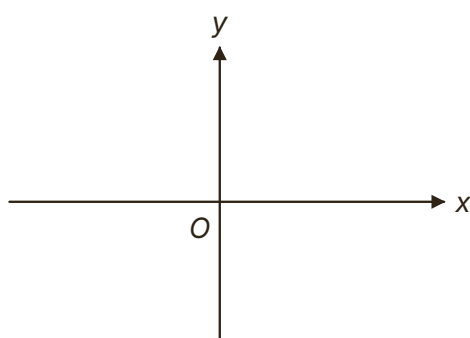
(a) Write down an expression for $\frac{dy}{dx}$.

[1 mark]

Answer

(b) Work out the coordinates of the stationary points and determine whether each is a maximum or a minimum. You must show your working. [5 marks]

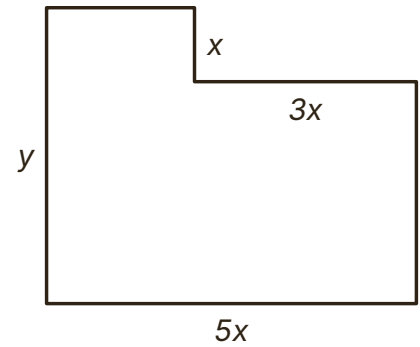
(c) Sketch the curve on the axes. Label the stationary points and the point where the curve crosses the y -axis. [2 marks]



Total for D2: 8 marks

D3 The diagram shows an L shape. It is a rectangle $5x$ cm by y cm with a rectangle $3x$ cm by x cm cut from one corner. The perimeter of the L shape is 224 cm.

- (a) Show that $y = 112 - 5x$. [2 marks]



Not drawn accurately

- (b) The area of the L shape is A cm². Show that $A = 560x - 28x^2$. [2 marks]

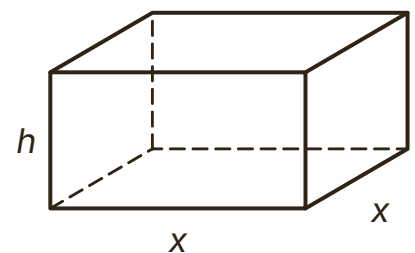
- (c) Use calculus to work out the maximum value of A as x varies. [3 marks]

Answer cm²

Total for D3: 7 marks

D4 An open box has no lid. Its base is a square of side x cm. Its height is h cm. Its volume is 256 cm³.

- (a) Show that the area of card used to make the box, A cm², is $A = x^2 + \frac{1024}{x}$. [3 marks]



Not drawn accurately

- (b) Work out the value of x that makes A as small as possible. Show that it gives a minimum. [4 marks]

Answer $x = \dots\dots$

- (c) Work out the height of the box when A is least. [1 mark]

Answer cm

Total for D4: 8 marks

- D5** $y = 2x + \frac{64}{x^2}$. Show that y has a minimum value when $x = 4$. Work out that minimum value. [5 marks]

Answer

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A metal tin is a cylinder with a base but no lid. Its volume is $64\pi \text{ cm}^3$. Work out the radius that uses the least metal, and the area of metal used. Give the area in terms of π . [6 marks]

Answer radius cm, area cm^2

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write the volume and the area of metal in terms of r and the height h . Use the volume to remove h .
- Hint 2** $h = \frac{64}{r^2}$, so the area is $\pi r^2 + 2\pi r \times \frac{64}{r^2} = \pi r^2 + \frac{128\pi}{r}$.
- Hint 3** $\frac{dA}{dr} = 2\pi r - \frac{128\pi}{r^2} = 0$ gives $r^3 = 64$, so $r = 4$. The second derivative is positive, so it is a minimum. The area is $16\pi + 32\pi = 48\pi \text{ cm}^2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can work out the coordinates of the stationary points on a curve. ☐ ☐ ☒
- I can use the second derivative, or the gradient on each side, to say if a point is a maximum or a minimum. ☐ ☐ ☒
- I can sketch a curve from its stationary points and its y-intercept. ☐ ☐ ☒
- I can use calculus to find the greatest or least value in a problem. ☐ ☐ ☒