



Tangents and normals

About 80 minutes No calculator 85 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Find the equation of the line with gradient -3 that passes through $(2, 5)$. Give it in the form $y = mx + c$.

Answer

A2 From chapter 5 [2 marks]

Work out the equation of the line through $(6, -1)$ that is perpendicular to $y = 2x + 7$. Give it in the form $ax + by + c = 0$.

Answer

- A3** Last lesson [2 marks]

Work out $\frac{dy}{dx}$ when $y = 4x^3 - \frac{6}{x}$.

Answer

A4 From chapter 4 [2 marks]

$(x + 2)$ is a factor of $x^3 + kx^2 + x + 6$. Work out the value of k .

Answer

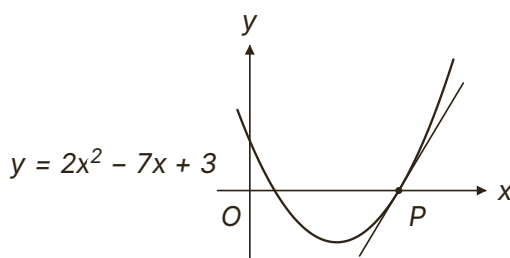
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the equation of the tangent to the curve $y = 2x^2 - 7x + 3$ at the point P where $x = 3$.



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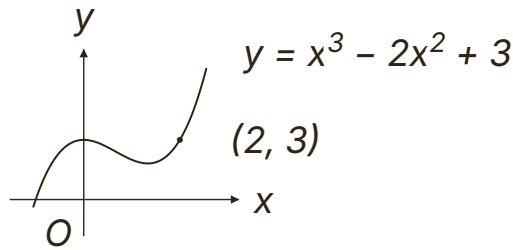
Hint Work out $\frac{dy}{dx}$ and put $x = 3$ in to get the gradient. You also need the y -coordinate of P .

- B1** [4 marks]
- Work out the equation of the tangent to the curve $y = x^3 - 4x + 1$ at the point P where $x = 2$. Give your answer in the form $y = mx + c$.

Answer $y = \dots\dots\dots$

EXAMPLE 2

Work out the equation of the normal to the curve $y = x^3 - 2x^2 + 3$ at the point $(2, 3)$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.



Not drawn accurately

Hint The normal is perpendicular to the tangent. Its gradient is -1 divided by the gradient of the tangent.

B2 Work out the equation of the normal to the curve [5 marks]

$y = 2x^2 + x - 6$ at the point $(-2, 0)$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Answer = 0

EXAMPLE 3

Work out the equation of the tangent to the curve $y = \frac{x^3 - 4}{x}$ at the point P where $x = 2$.

Hint Split the fraction first: $y = x^2 - 4x^{-1}$. Then differentiate each term.

B3 Work out the equation of the tangent to the curve [5 marks]

$y = \frac{x^3 + 6}{x^2}$ at the point P where $x = -1$. Give your answer in the form $y = mx + c$.

Answer $y = \dots\dots\dots$

EXAMPLE 4

Work out the coordinates of the points on the curve $y = x^3 - 6x^2 + 10x - 1$ where the tangent is parallel to the line $y = x + 4$. Hence work out the equation of each of these tangents.

Hint Parallel lines have equal gradients, so solve $\frac{dy}{dx} = 1$.

B4 Work out the coordinates of the two points on the curve [4 marks]

$y = x^3 + 3x^2 - 7x + 2$ where the tangent is parallel to the line $y = 2x - 5$.

Answer $(\dots\dots, \dots\dots)$ and $(\dots\dots, \dots\dots)$

EXAMPLE 5

The point P on the curve $y = ax^3 - 5x$ has x -coordinate 1. The normal to the curve at P has gradient $-\frac{1}{7}$. Work out the value of a .

Hint If the normal has gradient $-\frac{1}{7}$, the tangent has gradient 7, because the two gradients multiply to -1 .

- B5** The curve $y = x^2 + bx + c$ [4 marks]
 passes through the point $(1, 4)$. The normal to the curve at $(1, 4)$ has gradient $\frac{1}{3}$. Work out the values of b and c .

Answer $b = \dots\dots$ $c = \dots\dots$

c Practice

Level 2

- C1** Show that the line $y = 7x - 9$ is [3 marks]
 the tangent to the curve
 $y = x^3 - 5x + 7$ at the point
 $(2, 5)$.
- C2** The curve [4 marks]
 $y = (x + 1)(x - 2)(x - 3)$
 crosses the y -axis at A . Work
 out the equation of the tangent
 to the curve at A .

Answer $y = \dots\dots\dots$

- C3** Work out the equations of the [3 marks]
 two tangents to the curve
 $y = x^3 - 12x + 5$ that are parallel
 to the x -axis.
- C4** The point $P(2, 1)$ lies on the [4 marks]
 curve $y = x^2 - 5x + 7$. Q is the
 point on the curve where the
 normal is parallel to the tangent
 at P . Work out the coordinates
 of Q .

Answer $y = \dots\dots$ and $y = \dots\dots$

Answer $(\dots\dots, \dots\dots)$

- C5** The curve $y = x^2 - 8x + 12$ [5 marks]
crosses the x -axis at A and B .
The tangents to the curve at A
and at B meet at T . Work out the
coordinates of T .

Answer (.....,)

D Exam-style questions

AQA exam style

- D1** Ravi is finding the normal to the curve $y = x^2 + 5x - 2$ at the point $(1, 4)$. He writes: $\frac{dy}{dx} = 2x + 5 = 7$, so the normal is $y - 4 = 7(x - 1)$. [3 marks]

Explain Ravi's mistake. Work out the correct equation of the normal. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Answer = 0

- D2** A curve has equation $y = kx^3 - 6x$, where k is a constant. P is the point on the curve where $x = 2$. The normal to the curve at P is parallel to the line $x + 18y = 7$.

- (a) Work out the value of k . [4 marks]

Answer $k = \dots\dots$

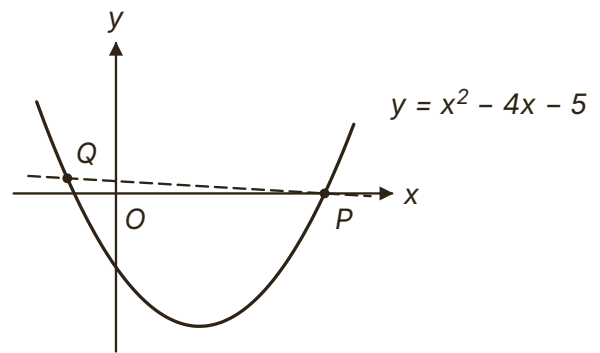
- (b) Hence work out the equation of the tangent to the curve at P . Give your answer in the form $y = mx + c$. [2 marks]

Answer $y = \dots\dots\dots$

Total for D2: 6 marks

D3 A curve has equation $y = x^2 - 4x - 5$. P is the point on the curve where $x = 5$.

- (a) Show that the equation of the normal to the curve at P is $x + 6y = 5$. [4 marks]



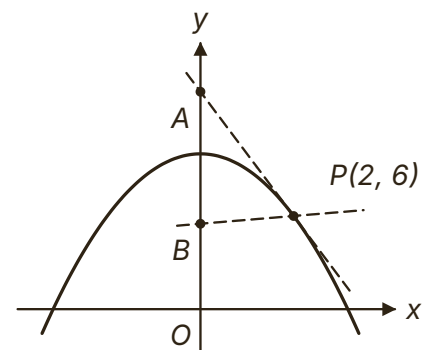
- (b) The normal at P meets the curve again at Q . Work out the x -coordinate of Q . You must show your working. [3 marks] Not drawn accurately

Answer $x = \dots\dots$

Total for D3: 7 marks

D4 The curve $y = 10 - x^2$ passes through the point $P(2, 6)$.

- (a) Work out the equation of the tangent to the curve at P . [3 marks]



Answer $y = \dots\dots\dots$

- (b) Work out the equation of the normal to the curve at P . [2 marks]

Not drawn accurately

Answer $\dots\dots\dots$

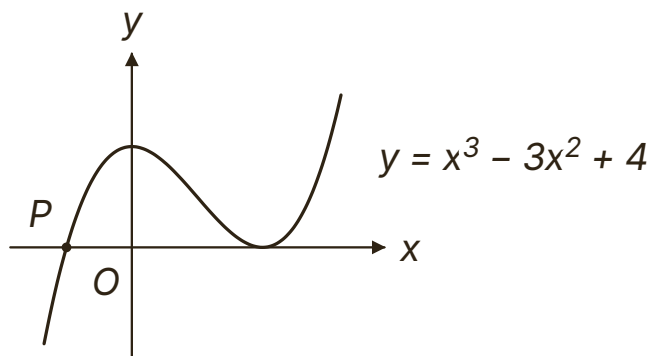
- (c) The tangent at P meets the y -axis at A . The normal at P meets the y -axis at B . Work out the area of triangle ABP . [3 marks]

Answer $\dots\dots$ square units

Total for D4: 8 marks

D5 The sketch shows the curve $y = x^3 - 3x^2 + 4$. The curve crosses the negative x -axis at $P(-1, 0)$.

- (a) Work out the equation of the tangent to the curve at P . [3 marks]



Answer $y = \dots\dots\dots$

- (b) The tangent to the curve at the point Q is parallel to the tangent at P . Work out the equation of the tangent at Q . [4 marks]

Not drawn accurately

Answer $y = \dots\dots\dots$

Total for D5: 7 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The tangent to the curve $y = x^3 - 2x + 1$ at the point P where $x = 2$ meets the curve again at Q . Work out the coordinates of Q . [5 marks]

Answer $(\dots\dots, \dots\dots)$

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Find the equation of the tangent at P first. Then solve the equations of the curve and the tangent together.
- Hint 2** P is $(2, 5)$ and $\frac{dy}{dx} = 3x^2 - 2 = 10$ there, so the tangent is $y = 10x - 15$.
- Hint 3** $x^3 - 2x + 1 = 10x - 15$ gives $x^3 - 12x + 16 = 0$. The tangent touches the curve at $x = 2$, so $(x - 2)^2$ is a factor: $x^3 - 12x + 16 = (x - 2)^2(x + 4)$. So $x = -4$ and $y = -64 + 8 + 1 = -55$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out the equation of the tangent to a curve at a point. ☐ ☐ ☐

I can work out the equation of the normal, in the form $ax + by + c = 0$. ☐ ☐ ☐

I can find the points where a tangent or normal has a given gradient. ☐ ☐ ☐

I can use tangents and normals to find unknown coefficients, crossing points and areas. ☐ ☐ ☐