

Tangents, chords and circles

About 85 minutes No calculator 78 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-tangents-chords-and-circles/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Work out the gradient of a line perpendicular to the line through $(1, -2)$ and $(5, 4)$.

A2 Needed today [2 marks]

Solve $x^2 + (x - 3)^2 = 29$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

- A3** Last lesson [2 marks]

Work out the centre and the radius of the circle $x^2 + y^2 - 4x + 10y + 13 = 0$.

A4 From chapter 4 [2 marks]

Work out the value of $b^2 - 4ac$ for $3x^2 - 5x + 4 = 0$. How many real solutions does the equation have?

Answer centre $(\dots, \dots)$ radius ...

Answer $\dots\dots$, $\dots\dots$ solutions

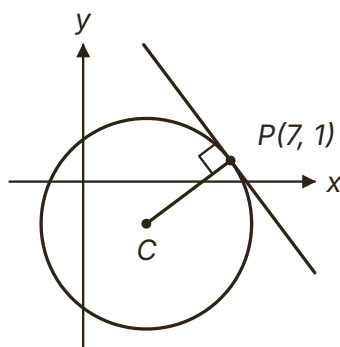
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$P(7, 1)$ lies on the circle $(x - 3)^2 + (y + 2)^2 = 25$. Work out the equation of the tangent to the circle at P . Give your answer in the form $ax + by = c$.



Not drawn accurately

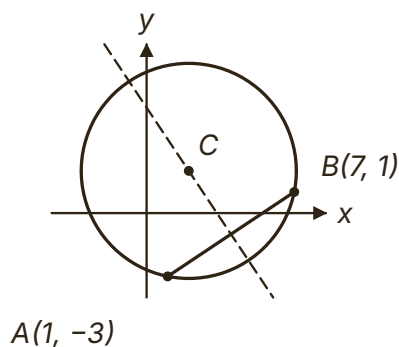
Hint The tangent is perpendicular to the radius CP .

- B1** $P(2, 3)$ lies on the circle $(x + 4)^2 + (y - 1)^2 = 40$. Work out the equation of the tangent to the circle at P . Give your answer in the form $y = mx + c$. [3 marks]

Answer

EXAMPLE 2

$A(1, -3)$ and $B(7, 1)$ are points on a circle with centre C . Work out the equation of the line through C that is perpendicular to AB . Give your answer in the form $ax + by = c$.



Not drawn accurately

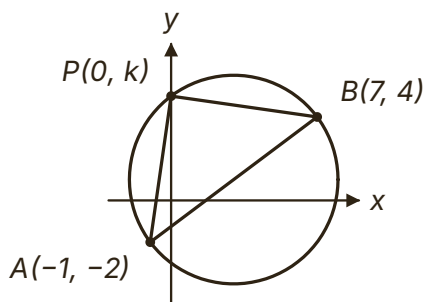
Hint The perpendicular from the centre to a chord passes through the midpoint of the chord.

B2 $A(-3, 2)$ and $B(5, 6)$ are points on a circle. The centre of the circle lies on the x -axis. Work out the coordinates of the centre. [4 marks]

Answer (.....,)

EXAMPLE 3

AB is a diameter of a circle. A is $(-1, -2)$ and B is $(7, 4)$. $P(0, k)$ is a point on the circle, with $k > 0$. Work out the value of k .



Not drawn accurately

Hint The angle in a semicircle is 90° , so AP and BP are perpendicular: their gradients multiply to -1 .

B3 AB is a diameter of a circle. A is $(-4, 1)$ and B is $(2, 5)$. $P(k, 0)$ is a point on the circle, with $k > 0$. Work out the value of k . [4 marks]

Answer $k = \dots\dots$

EXAMPLE 4

Work out the coordinates of the points where the line $y = 2x - 1$ meets the circle $x^2 + y^2 = 13$.

Hint Put $y = 2x - 1$ into the equation of the circle and solve the quadratic.

B4 Work out the coordinates of the points where the line [4 marks]

$x + 2y = 9$ meets the circle $(x - 2)^2 + (y - 1)^2 = 25$.

Answer (... , ...) and (... , ...)

EXAMPLE 5

Show that the line $x + y = 6$ is a tangent to the circle $x^2 + y^2 = 18$. Work out the coordinates of the point where it touches the circle.

Hint A line that touches a circle meets it at one point only: the quadratic has one repeated root.

B5 Show that the line $y = 2x - 5$ is a tangent to the circle [4 marks]

$(x - 1)^2 + (y - 2)^2 = 5$. Work out the coordinates of the point where it touches the circle.

Answer (.....,)

c Practice

Level 2

C1 $P(-2, 6)$ lies on the circle $x^2 + y^2 = 40$. Work out the equation of the tangent to the circle at P . Give your answer in the form $ax + by = c$. [3 marks]

Answer

C2 A circle has centre $C(-3, 2)$. The chord PQ has midpoint $M(0, 5)$. Work out the equation of the line PQ . [3 marks]

Answer

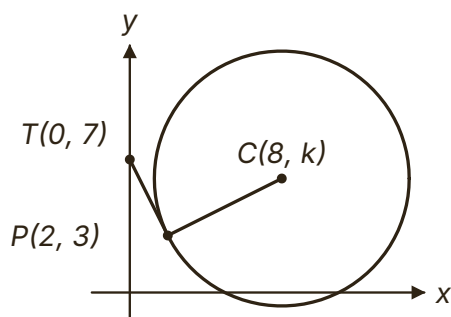
- C3** T is the point $(9, 5)$. A tangent is drawn from T to the circle $(x - 1)^2 + (y + 1)^2 = 25$. Work out the length of the tangent. Give your answer as a surd in its simplest form. [3 marks]

- C4** Show that the line $y = x - 6$ does not meet the circle $x^2 + y^2 = 12$. [3 marks]

Answer

- C5** A circle has centre $C(8, k)$. The line PT is a tangent to the circle at $P(2, 3)$. T is the point $(0, 7)$. Work out the value of k . [3 marks]

- C6** The line $x + 2y = 15$ is a tangent to the circle $x^2 + y^2 = 45$ at the point P . Work out the coordinates of P . [3 marks]



Not drawn accurately

Answer (.....,)

Answer $k = \dots\dots$

D Exam-style questions

AQA exam style

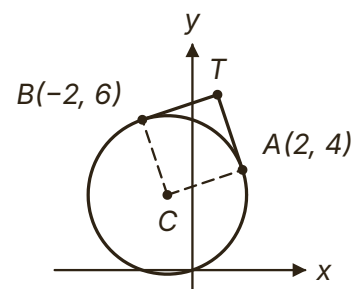
- D1** Circle the gradient of the tangent to the circle $x^2 + y^2 = 29$ at the point $(5, -2)$. [1 mark]
 $-\frac{2}{5}$ $\frac{5}{2}$ $-\frac{5}{2}$ $\frac{2}{5}$

Answer

- D2** Ben says the line $y = 2x - 5$ is a tangent to the circle $x^2 + y^2 = 10$, because it passes through the point $(3, 1)$, which is on the circle. [3 marks]
 Show that Ben is wrong.

D3 The circle $(x + 1)^2 + (y - 3)^2 = 10$ has centre C . $A(2, 4)$ and $B(-2, 6)$ are points on the circle. The tangents at A and B meet at T .

- (a) Work out the equation of the tangent at A . [3 marks]
Give your answer in the form $y = mx + c$.



Answer

- (b) The tangent at B has equation $x - 3y + 20 = 0$. [2 marks]
Work out the coordinates of T .

Not drawn accurately

Answer (.....,)

- (c) Work out the area of the quadrilateral $CATB$. [3 marks]

Answer units²

Total for D3: 8 marks

D4 The line $y = x + 1$ meets the circle $(x - 3)^2 + (y - 2)^2 = 20$ at the points P and Q . [5 marks]
Work out the length of PQ . Give your answer as a surd in its simplest form. You must show your working.

Answer

D5 The circle $(x - 4)^2 + (y - 7)^2 = 13$ has centre C . $P(6, 4)$ is a point on the circle. O is the origin.

- (a) Show that OP is a tangent to the circle. [3 marks]

(b) Work out the length of OP . Give your answer as a surd in its simplest form. [2 marks]

Answer

Total for D5: 5 marks

D6 The line $y = x + c$ is a tangent to the circle $(x - 2)^2 + (y + 1)^2 = 32$. Work out the two possible values of c . You must show your working. [5 marks]

Answer $c = \dots\dots$ or $c = \dots\dots$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Two tangents to the circle $(x - 6)^2 + (y - 2)^2 = 8$ pass through the origin. Work out the equation of each tangent. [6 marks]

Answer and

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** A line through the origin is $y = mx$. Put it into the equation of the circle.
- Hint 2** $(1 + m^2)x^2 - (12 + 4m)x + 32 = 0$. The line is a tangent when this has one repeated root, so $b^2 - 4ac = 0$.
- Hint 3** $(12 + 4m)^2 - 128(1 + m^2) = 0$ gives $7m^2 - 6m - 1 = 0$, so $(7m + 1)(m - 1) = 0$: $m = 1$ or $m = -\frac{1}{7}$. The tangents are $y = x$ and $y = -\frac{1}{7}x$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the equation of the tangent to a circle at a point on it. ☐ ☐ ☒

I can use "the perpendicular from the centre bisects a chord" and "the angle in a semicircle is 90° " with coordinates. ☐ ☐ ☒

I can find where a line meets a circle, and show that a line touches it or misses it. ☐ ☐ ☒

I can work out the length of a tangent from a point outside a circle. ☐ ☐ ☒