



The area of a triangle

About 80 minutes Calculator allowed 68 marks

Date _____

A Do Now

Four questions to start. Two of them use what you need today.

- A1** Needed today [2 marks]

Triangle ABC has a right angle at C . $AB = 14$ cm and angle $BAC = 37^\circ$. Work out the length of BC . Give your answer to 3 significant figures.

A2 Last lesson [2 marks]

Do not use a calculator.
 $\sin x = \frac{2}{3}$ and $90^\circ < x < 180^\circ$.
Work out the exact value of $\cos x$.

Answer cm

Answer $\cos x = \dots\dots$

- A3** Needed today [2 marks]

Solve $\sin x = 0.8$ for $0^\circ \leq x \leq 180^\circ$. Give your answers to 1 decimal place.

A4 GCSE Higher [2 marks]

A sector of a circle has radius 6 cm and angle 50° . Work out its area. Give your answer to 3 significant figures.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer cm²

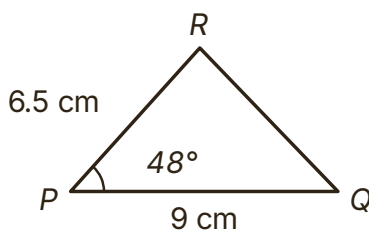
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Area of a triangle = $\frac{1}{2}ab \sin C$, where C is the angle between the sides a and b . Give answers to 3 significant figures.

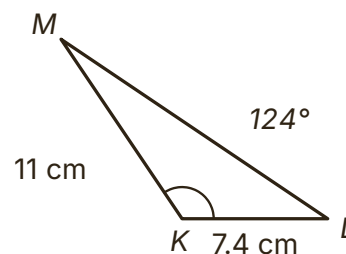
EXAMPLE 1

Work out the area of triangle PQR .



Hint The 48° angle is between the two sides you know, so use $\frac{1}{2}ab \sin C$ straight away.

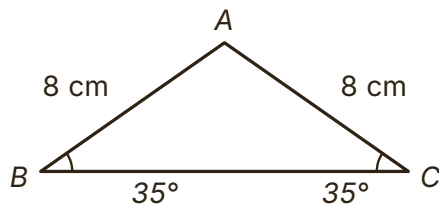
- B1** Work out the area of triangle KLM . [2 marks]



Answer cm²

EXAMPLE 2

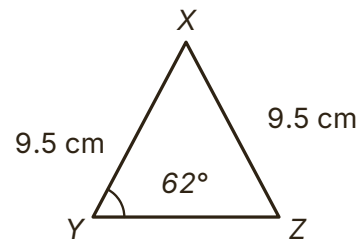
Triangle ABC is isosceles with $AB = AC = 8$ cm and angle $ABC = 35^\circ$. Work out its area.



Hint You need the angle BETWEEN the two 8 cm sides. That is angle $BAC = 180 - 35 - 35$.

B2 Triangle XYZ is isosceles [3 marks]

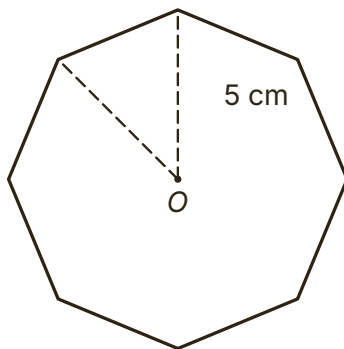
with $XY = XZ = 9.5$ cm and angle $XYZ = 62^\circ$. Work out its area.



Answer cm^2

EXAMPLE 3

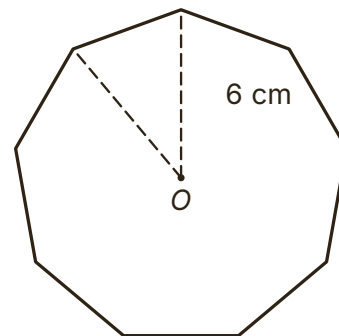
A regular octagon is drawn inside a circle of radius 5 cm, with its corners on the circle. Work out the area of the octagon.



Hint Join each corner to the centre. This makes 8 equal triangles, each with two sides of 5 cm and an angle of $360 \div 8$ at the centre.

B3 A regular nine-sided polygon [3 marks]

is drawn inside a circle of radius 6 cm, with its corners on the circle. Work out the area of the polygon.



Answer cm^2

EXAMPLE 4

Triangle ABC has area 30 cm^2 . $CA = 8 \text{ cm}$ and $CB = 10 \text{ cm}$. Work out the two possible sizes of angle ACB . Give your answers to 1 decimal place.

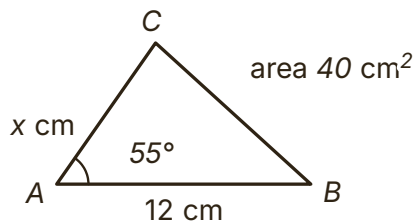
Hint Find $\sin C$ from the area. Then remember that $\sin(180^\circ - C) = \sin C$: an obtuse angle gives the same area.

- B4** Triangle PQR has area 18 cm^2 . [3 marks]
 $PQ = 6 \text{ cm}$ and $PR = 9 \text{ cm}$.
 Work out the two possible sizes of angle QPR . Give your answers to 1 decimal place.

Answer $^\circ$ or $^\circ$

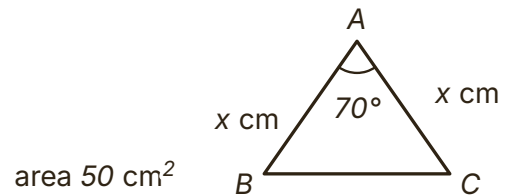
EXAMPLE 5

Triangle ABC has area 40 cm^2 . $AB = 12 \text{ cm}$ and angle $BAC = 55^\circ$. Work out the length AC .



Hint Write the area formula with $AC = x$, then solve for x .

- B5** An isosceles triangle has area 50 cm^2 . The angle between its two equal sides is 70° . Work out the length of one of the equal sides.



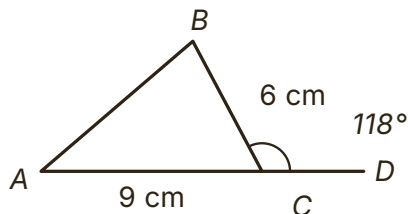
Not drawn accurately

Answer cm

c Practice

Level 2

- C1** A , C and D lie on a straight line. [3 marks]
 Work out the area of triangle ABC .



- C2** A regular hexagon has sides of length 9 cm . Work out its area. [3 marks]

Answer cm^2

Answer cm^2

- C3** $ABCD$ is a rhombus with sides of 6.5 cm and angle $DAB = 72^\circ$. Work out its area. [3 marks]

Answer cm^2

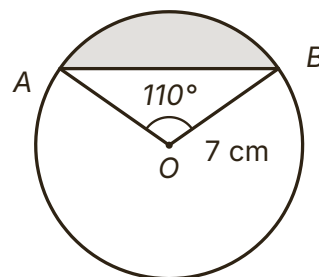
- C4** Do not use a calculator. In triangle DEF , $DE = 6$ cm, $DF = 8\sqrt{2}$ cm and angle $EDF = 135^\circ$. Work out the area of the triangle. Give your answer as an integer. [3 marks]

Answer cm^2

- C5** A regular tetrahedron has four faces. Each face is an equilateral triangle with sides of 8 cm. Work out the total surface area of the tetrahedron. [3 marks]

Answer cm^2

- C6** The circle has centre O and radius 7 cm. Angle $AOB = 110^\circ$. Work out the area of the shaded segment. [4 marks]



Answer cm^2

D Exam-style questions

AQA exam style

- D1** Sam says, "I drew a triangle with two sides of 10 cm and 6 cm. Its area is 35 cm^2 ." [2 marks]

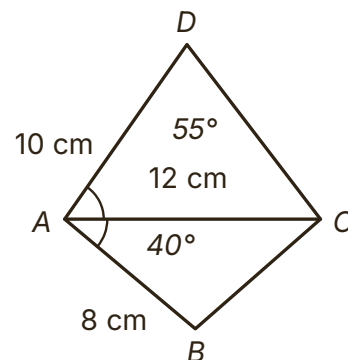
Show that Sam must be wrong.

- D2** A regular hexagon has an area of 150 cm^2 . Work out the length of one side. Give your answer to 3 significant figures. [3 marks]

Answer cm

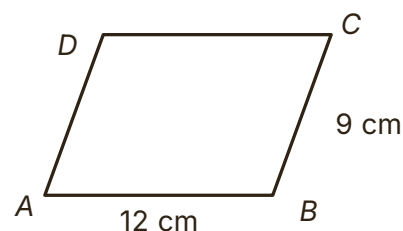
- D3** The quadrilateral $ABCD$ is made from two triangles. Work out the area of $ABCD$. [4 marks]
Give your answer to 3 significant figures.

Answer cm^2



- D4** $ABCD$ is a parallelogram. $AB = 12 \text{ cm}$ and $BC = 9 \text{ cm}$. The area of $ABCD$ is 86.4 cm^2 . Angle ABC is obtuse.

- (a) Show that $\sin ABC = 0.8$ [2 marks]



Not drawn accurately

- (b) Work out the size of angle ABC . Give your answer to 1 decimal place. [2 marks]

Answer $^\circ$

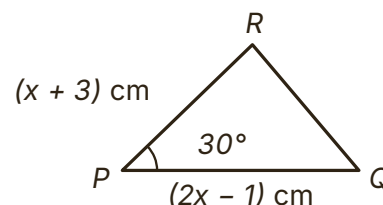
- (c) Write down the area of triangle ABD . [1 mark]

Answer cm^2

Total for D4: 5 marks

- D5** Do not use a calculator. In triangle PQR , $PQ = (2x - 1) \text{ cm}$, $PR = (x + 3) \text{ cm}$ and angle $QPR = 30^\circ$. The area of the triangle is 18 cm^2 .

- (a) Show that $2x^2 + 5x - 75 = 0$ [3 marks]



Not drawn accurately

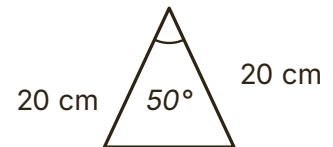
- (b) Hence work out the length of PQ . [2 marks]

Answer cm

Total for D5: 5 marks

D6 A tiler covers a wall of area 3 m^2 with triangular tiles. Each tile is an isosceles triangle with two sides of 20 cm and an angle of 50° between them.

- (a) Work out the area of one tile. Give your answer to 3 significant figures. [2 marks]



Answer cm^2

- (b) The tiler divides the area of the wall by the area of one tile and rounds up. What number does this give? [1 mark]

Answer

- (c) Give one reason why the tiler is likely to need more tiles than this. [1 mark]

Total for D6: 4 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Two sides of a triangle have lengths that add up to 20 cm. The angle between them is 50° . Work out the greatest possible area of the triangle. Give your answer to 3 significant figures. [4 marks]

Answer cm^2

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call one side x cm. The other side is $(20 - x)$ cm.
Hint 2 Area = $\frac{1}{2}x(20 - x) \sin 50^\circ$. Only $x(20 - x)$ changes as x changes.
Hint 3 $x(20 - x) = 100 - (x - 10)^2$, which is greatest when $x = 10$. Area = $\frac{1}{2} \times 10 \times 10 \times \sin 50^\circ = 38.3 \text{ cm}^2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out the area of any triangle from two sides and the angle between them. ☐ ☐ ☐

I can find the area of a regular polygon or a segment by splitting it into triangles. ☐ ☐ ☐

I can work backwards from an area to a side, or to the two possible angles. ☐ ☐ ☐