

The cosine rule, and using both rules

About 90 minutes Calculator allowed 65 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-the-cosine-rule-and-using-both-rules/

Date _____

A Do Now

Answer these before the lesson starts.

- A1** Needed today [2 marks]

Solve $x^2 - 5x - 14 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

A2 Needed today [2 marks]

Solve $\cos \theta = -0.35$ for $0^\circ < \theta < 180^\circ$. Give your answer to 1 decimal place.

Answer $\theta = \dots\dots^\circ$

- A3** Last lesson [2 marks]

In triangle ABC , angle $A = 52^\circ$, angle $B = 71^\circ$ and $BC = 8.6$ cm. Work out AC to 3 significant figures.

Answer $AC = \dots\dots$ cm

A4 From chapter 7 [2 marks]

A triangle has sides of 6 cm and 10 cm with an angle of 150° between them. Work out its exact area.

Answer $\dots\dots$ cm²

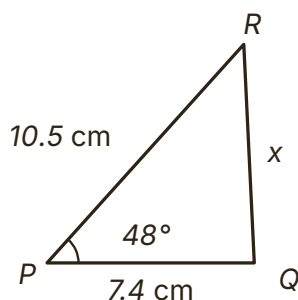
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Give lengths and angles to 3 significant figures unless the question says otherwise.

EXAMPLE 1

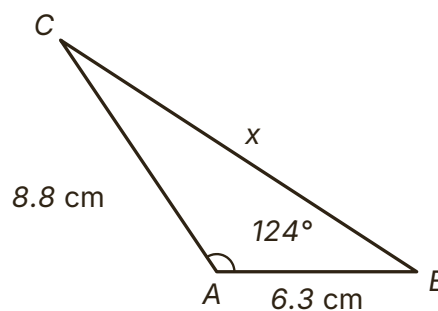
Work out the length x .



Not drawn accurately

Hint You know two sides and the angle between them, so use $a^2 = b^2 + c^2 - 2bc \cos A$. Remember to take the square root at the end. If the angle is obtuse, $\cos A$ is negative and the last term is added.

- B1** Work out the length x . [3 marks]

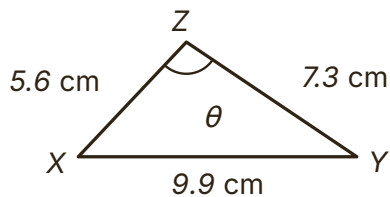


Not drawn accurately

Answer $x = \dots\dots$ cm

EXAMPLE 2

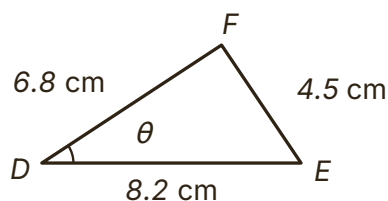
Work out the size of angle θ .



Not drawn accurately

Hint You know three sides, so use

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, where a is the side opposite the angle you want. The largest angle is opposite the longest side.

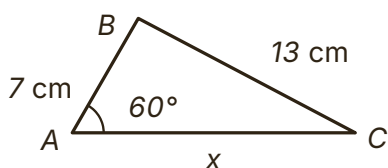
B2 Work out the size of angle θ . [3 marks]

Not drawn accurately

Answer $\theta = \dots\dots^\circ$

EXAMPLE 3

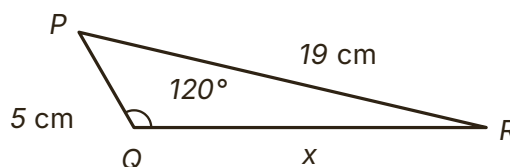
Work out the length x . Do not use a calculator.



Not drawn accurately

Hint The unknown x is next to the 60° angle, so put x into the cosine rule for the side opposite:

$13^2 = x^2 + 7^2 - 2 \times x \times 7 \times \cos 60^\circ$. This gives a quadratic.

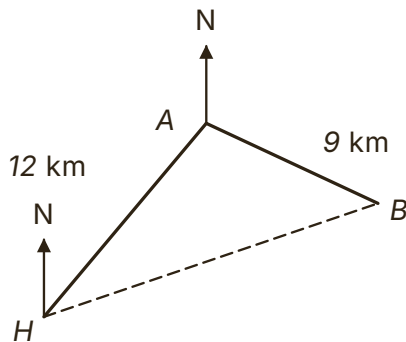
B3 Work out the length x . Do not use a calculator. [4 marks]

Not drawn accurately

Answer $x = \dots\dots$ cm

EXAMPLE 4

A ship sails 12 km from harbour H on a bearing of 040° to A . It then sails 9 km on a bearing of 115° to B . Work out the distance HB and the bearing of B from H .

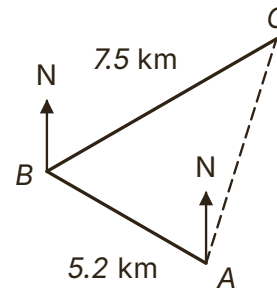


Not drawn accurately

Hint At A , the bearing back to H is 220° , so angle $HAB = 220^\circ - 115^\circ$. Use the cosine rule for HB , then the sine rule for angle AHB .

B4 [5 marks]

Mia walks 5.2 km from A on a bearing of 300° to B . She then walks 7.5 km on a bearing of 060° to C . Work out the distance AC and the bearing of C from A .



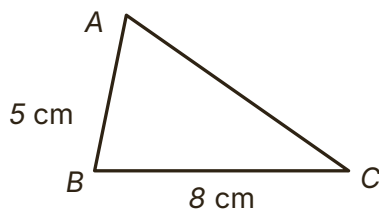
Not drawn accurately

Answer $AC = \dots\dots$ km, bearing $\dots\dots^\circ$

c Practice

Level 2

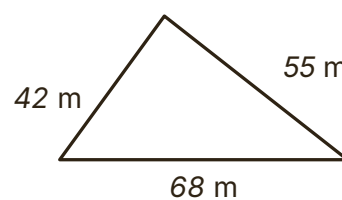
- C1** In triangle ABC , $AB = 5$ cm, $BC = 8$ cm and $\cos B = \frac{1}{5}$. Work out the exact length of AC . [3 marks]



Not drawn accurately

Answer $AC = \dots\dots$ cm

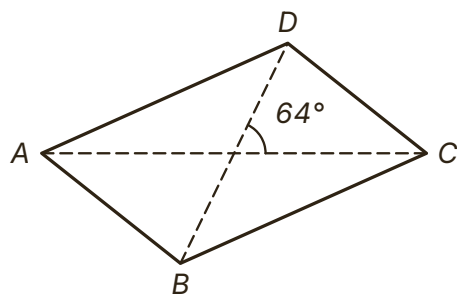
- C2** A field is a triangle with sides of 42 m, 55 m and 68 m. Work out its area to the nearest square metre. [4 marks]



Not drawn accurately

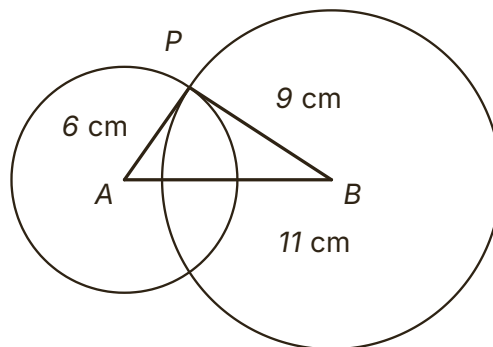
Answer $\dots\dots$ m²

- C3** $ABCD$ is a parallelogram. The diagonals $AC = 22$ cm and $BD = 14$ cm meet at O . The angle between the diagonals is 64° . Work out the lengths of the sides of the parallelogram. [4 marks]



Not drawn accurately

- C4** A circle with centre A and radius 6 cm and a circle with centre B and radius 9 cm meet at P . $AB = 11$ cm. Work out the size of angle PAB . [3 marks]



Not drawn accurately

Answer angle $PAB = \dots\dots^\circ$

Answer $\dots\dots$ cm and $\dots\dots$ cm

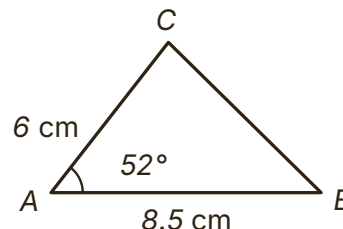
D Exam-style questions

AQA exam style

- D1** In triangle ABC , $AB = 8.5$ cm, $AC = 6$ cm and angle $BAC = 52^\circ$. Sam works out BC like this: " $BC^2 = (8.5^2 + 6^2 - 2 \times 8.5 \times 6) \times \cos 52^\circ = 3.85$, so $BC = 1.96$ cm."

- (a) Explain the mistake Sam has made. [1 mark]

- (b) Work out the length of BC . [2 marks]



Not drawn accurately

Answer $BC = \dots\dots$ cm

- (c) Work out the size of angle ABC . [2 marks]

Answer $\dots\dots^\circ$

Total for D1: 5 marks

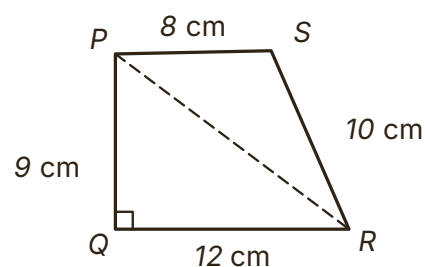
D2 $PQRS$ is a quadrilateral. Angle $PQR = 90^\circ$, $PQ = 9$ cm, $QR = 12$ cm, $RS = 10$ cm and $SP = 8$ cm.

(a) Show that $PR = 15$ cm.

[1 mark]

(b) Work out the size of angle PSR .

[3 marks]



Not drawn accurately

Answer $^\circ$

(c) Work out the area of $PQRS$.

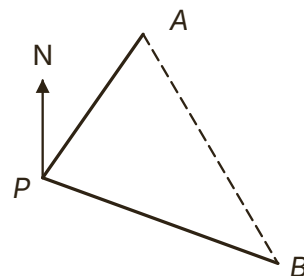
[3 marks]

Answer cm^2

Total for D2: 7 marks

D3 Two boats leave port P at 12 noon. Boat A sails on a bearing of 035° at 14 km/h. Boat B sails on a bearing of 110° at 20 km/h.

(a) Work out the distance between the boats at 1.30 pm. [3 marks]



Answer km

Not drawn accurately

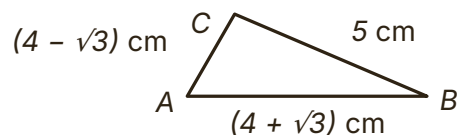
(b) Work out the bearing of B from A at 1.30 pm. You must show your working. [3 marks]

Answer $^\circ$

Total for D3: 6 marks

D4 In triangle ABC , $AB = (4 + \sqrt{3})$ cm, $AC = (4 - \sqrt{3})$ cm and $BC = 5$ cm. Do not use a calculator in this question.

- (a) Show that angle $BAC = 60^\circ$. [4 marks]



Not drawn accurately

- (b) Work out the area of the triangle. Give your answer in the form $\frac{a\sqrt{3}}{b}$ where a and b are integers. [2 marks]

Answer cm²

Total for D4: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** In triangle ABC , $AB = 7$ cm, $AC = 9$ cm and $BC = 10$ cm. M is the midpoint of BC . Work out the exact length of AM . [4 marks]

Answer $AM = \dots$ cm

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** AM is a side of triangle ABM . You know AB and BM ; you need the angle at B .
- Hint 2** In triangle ABC , $\cos B = \frac{7^2 + 10^2 - 9^2}{2 \times 7 \times 10} = \frac{68}{140}$.
- Hint 3** In triangle ABM ,
 $AM^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \frac{68}{140} = 74 - 34 = 40$, so
 $AM = \sqrt{40} = 2\sqrt{10}$ cm.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can use the cosine rule to find a side when I know two sides and the angle between them. ☐ ☐ ☐
- I can use the cosine rule to find an angle when I know three sides. ☐ ☐ ☐
- I can decide whether to use the sine rule or the cosine rule. ☐ ☐ ☐
- I can use both rules in bearings, area and multi-step problems. ☐ ☐ ☐