



The equation of a circle

About 80 minutes No calculator 69 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Work out the distance between the points $(-2, 5)$ and $(4, -3)$.

A2 Needed today [2 marks]

Write $x^2 - 10x + 7$ in the form $(x - p)^2 - q$.

Answer

Answer

- A3** Last lesson [2 marks]

A is $(-4, 1)$ and B is $(8, 10)$. C is the point on AB with $AC : CB = 2 : 1$. Work out the coordinates of C .

A4 From chapter 1 [2 marks]

Write $\sqrt{80}$ in the form $a\sqrt{b}$, where b is as small as possible.

Answer

Answer $(\dots, \dots)$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

(a) Write down the equation of the circle with centre $(3, -4)$ and radius 6. (b) Write down the centre and the radius of the circle $x^2 + (y + 2)^2 = 49$.

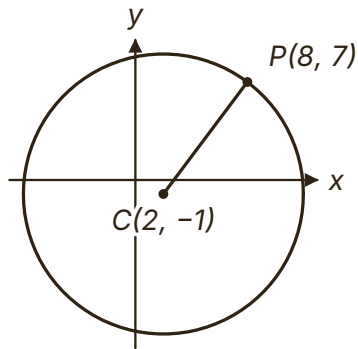
Hint A circle with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$. Watch the signs.

- B1** (a) Write down the equation of the circle with centre $(-5, 2)$ and radius $3\sqrt{2}$. (b) Write down the centre and the radius of the circle $(x - 6)^2 + y^2 = 28$. Give the radius as a surd in its simplest form. [3 marks]

Answer (a) (b) $(\dots, \dots)$, $r = \dots$

EXAMPLE 2

A circle has centre $C(2, -1)$ and passes through the point $P(8, 7)$. Work out the equation of the circle.



Not drawn accurately

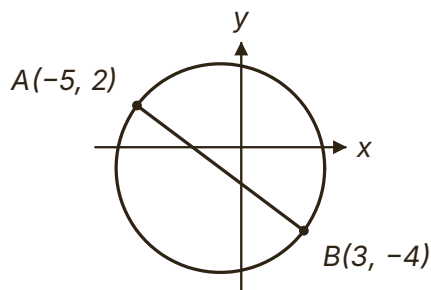
Hint The radius is the distance CP .

B2 A circle has centre $(-3, 5)$ and passes through the point $(2, 17)$. Work out the equation of the circle. [3 marks]

Answer

EXAMPLE 3

$A(-5, 2)$ and $B(3, -4)$ are the ends of a diameter of a circle. Work out the equation of the circle.



Not drawn accurately

Hint The centre is the midpoint of AB . The radius is half of AB , or the distance from the centre to A .

B3 $P(-6, -3)$ and $Q(4, 7)$ are the ends of a diameter of a circle. Work out the equation of the circle. [4 marks]

Answer

EXAMPLE 4

Show that $x^2 + y^2 + 6x - 4y - 12 = 0$ is the equation of a circle. Work out its centre and its radius.

Hint Complete the square in x and in y separately.

B4 A circle has equation [3 marks]

$$x^2 + y^2 - 12x + 4y + 4 = 0.$$

Work out the coordinates of its centre and its radius.

Answer centre $(\dots, \dots)$ radius $\dots\dots$

EXAMPLE 5

A circle has equation $(x - 2)^2 + (y + 1)^2 = 40$. Is each point inside, on or outside the circle?

$A(8, 1)$, $B(-3, 4)$, $C(4, 4)$

Hint Put each point into the left-hand side. Equal to 40: on. Less than 40: inside. More than 40: outside.

B5 A circle has equation [3 marks]

$$(x + 2)^2 + (y - 1)^2 = 34.$$

Is each point inside, on or outside the circle? Show your working. $P(3, 4)$, $Q(1, -5)$, $R(-5, 4)$

Answer $P \dots\dots$ $Q \dots\dots$ $R \dots\dots$

c Practice

Level 2

C1 A circle has centre $(5, -2)$. The [2 marks]
area of the circle is 45π . Work
out the equation of the circle.

Answer $\dots\dots\dots$

C2 A circle of radius 10 passes [4 marks]
through the points $(-1, 0)$ and
 $(11, 0)$. There are two possible
circles. Work out the equation of
each one.

Answer $\dots\dots\dots$ and $\dots\dots\dots$

C3 Show that [2 marks]
 $x^2 + y^2 - 6x + 10y + 40 = 0$ is
not the equation of a circle.

C4 The point $(a, 2)$ lies on the circle [3 marks]
 $(x - 1)^2 + (y + 3)^2 = 50$. Work
out the two possible values of a .

Answer $a = \dots\dots$ or $a = \dots\dots$

- C5** Work out the coordinates of every point where the circle $(x - 4)^2 + (y + 1)^2 = 17$ meets the x -axis or the y -axis. [4 marks]

Answer

D Exam-style questions

AQA exam style

- D1** Circle the radius of the circle $x^2 + (y + 3)^2 = 18$. 18 9 $3\sqrt{2}$ 3 [1 mark]

Answer

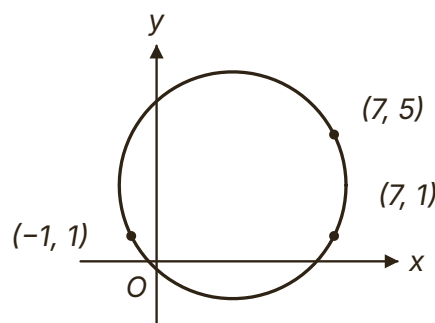
- D2** Sam says the circle $x^2 + y^2 - 8x + 2y = 8$ has centre $(-4, 1)$ and radius $\sqrt{8}$. [3 marks]

Explain Sam's two mistakes. Work out the correct centre and radius.

Answer centre $(\dots, \dots)$ radius

- D3** A circle passes through the points $(-1, 1)$, $(7, 1)$ and $(7, 5)$. Work out the equation of the circle. [4 marks]

Answer



Not drawn accurately

- D4** A circle has equation $x^2 + y^2 + 4x + 2y - 20 = 0$.

- (a) Work out the coordinates of the centre and the radius of the circle. [3 marks]

Answer centre $(\dots, \dots)$ radius

- (b) P is the point $(10, 4)$. Show that P is outside the circle, and work out the shortest distance from P to the circle. [3 marks]

Answer

Total for D4: 6 marks

- D5** A circle has centre $(1, 3)$ and passes through the point $(4, -2)$. The point (a, a) lies on the circle. Work out the two possible values of a . You must show your working. [5 marks]

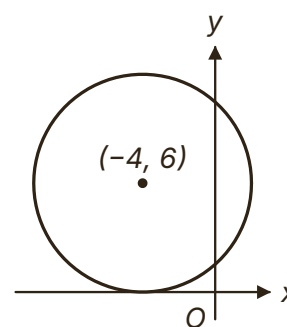
Answer $a = \dots\dots$ or $a = \dots\dots$

- D6** A circle has centre $(-4, 6)$. The x -axis is a tangent to the circle.

- (a) Write down the equation of the circle. [2 marks]

Answer

- (b) The circle crosses the y -axis at two points. Work out their exact coordinates. [3 marks]



Not drawn accurately

Answer and

Total for D6: 5 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A circle touches both the x -axis and the y -axis. It passes through the point $(4, 2)$. There are two possible circles. Work out the equation of each one. [6 marks]

Answer and

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** A circle touching both axes, with $(4, 2)$ in the first quadrant, has centre (r, r) and radius r .
- Hint 2** Put $(4, 2)$ into $(x - r)^2 + (y - r)^2 = r^2$.
- Hint 3** $(4 - r)^2 + (2 - r)^2 = r^2$ gives $r^2 - 12r + 20 = 0$, so $r = 2$ or $r = 10$. The circles are $(x - 2)^2 + (y - 2)^2 = 4$ and $(x - 10)^2 + (y - 10)^2 = 100$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the equation of a circle from its centre and radius, and read them back. ☐ ☐ ☐

I can complete the square to find the centre and radius of a circle. ☐ ☐ ☐

I can find the equation of a circle through given points or from a diameter. ☐ ☐ ☐

I can tell whether a point is inside, on or outside a circle. ☐ ☐ ☐