



The factor theorem

About 85 minutes No calculator 82 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks]

Expand and simplify $(x - 3)(x^2 + 2x - 4)$.

A2 GCSE Higher [2 marks]

Solve $x^2 - 4x - 3 = 0$. Give your answers in surd form.

Answer

Answer $x = \dots$

- A3** Last lesson [2 marks]

Solve the simultaneous equations $y = 2x - 1$ and $xy = 15$.

A4 From chapter 2 [2 marks]

Factorise $6x^2 + x - 12$.

Answer

Answer $(\dots, \dots)$ and $(\dots, \dots)$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = x^3 - 6x^2 + 5x + 12$. Work out $f(1)$, $f(-1)$ and $f(3)$. Hence factorise $f(x)$ fully and solve $f(x) = 0$.

Hint If $f(a) = 0$ then $(x - a)$ is a factor. Only factors of the constant term 12 are worth trying.

- B1** Show that $(x - 2)$ is a factor of $x^3 - 3x^2 - 10x + 24$. Hence factorise $x^3 - 3x^2 - 10x + 24$ fully. [4 marks]

Answer

EXAMPLE 2

$(x + 3)$ is a factor of $x^3 + x^2 - 4x + 6$. Work out the quadratic factor. Show that $(x + 3)$ is the only linear factor.

Hint Write $x^3 + x^2 - 4x + 6 \equiv (x + 3)(x^2 + bx + 2)$: the x^2 and the 2 must be there to make x^3 and 6. Then compare the x^2 terms.

EXAMPLE 3

Show that $(3x - 2)$ is a factor of $3x^3 + 4x^2 - 13x + 6$. Hence solve $3x^3 + 4x^2 - 13x + 6 = 0$.

Hint $(3x - 2)$ is zero when $x = \frac{2}{3}$, so work out $f\left(\frac{2}{3}\right)$. Then $3x^3 + 4x^2 - 13x + 6 \equiv (3x - 2)(x^2 + bx - 3)$.

EXAMPLE 4

$f(x) = x^3 + ax^2 + bx - 12$. $(x + 2)$ and $(x - 3)$ are factors of $f(x)$. Work out a and b , and the third linear factor.

Hint $f(-2) = 0$ and $f(3) = 0$ give two equations in a and b . The constants in the three brackets multiply to give -12 .

B2 $(x - 3)$ is a factor of $x^3 - 4x^2 + 6x - 9$. Show that the equation $x^3 - 4x^2 + 6x - 9 = 0$ has exactly one real root. [4 marks]

B3 Use the factor theorem to show that $(2x + 3)$ is a factor of $2x^3 + 5x^2 - x - 6$. Hence solve $2x^3 + 5x^2 - x - 6 = 0$. [5 marks]

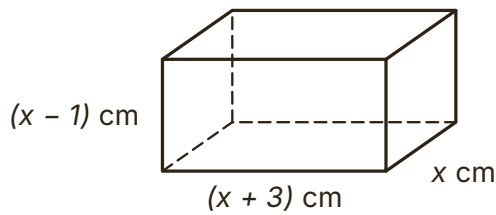
Answer $x = \dots\dots, \dots\dots, \dots\dots$

B4 $f(x) = x^3 + ax^2 + bx + 30$. $(x - 2)$ and $(x + 3)$ are factors of $f(x)$. Work out the values of a and b . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

EXAMPLE 5

A cuboid measures x cm by $(x + 3)$ cm by $(x - 1)$ cm. Its volume is 84 cm^3 . Show that $x^3 + 2x^2 - 3x - 84 = 0$. Hence work out the dimensions of the cuboid.

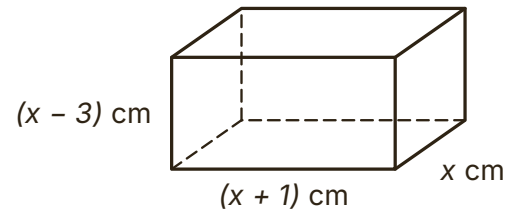


Not drawn accurately

Hint Multiply the three edges and set the product equal to 84. Try factors of 84 that make $x - 1$ positive.

B5 A cuboid measures x cm by $(x + 1)$ cm by $(x - 3)$ cm. Its volume is 126 cm^3 . Show that $x^3 - 2x^2 - 3x - 126 = 0$. [5 marks]

Hence work out the dimensions of the cuboid.



Not drawn accurately

Answer cm by cm by cm

c Practice

Level 2

C1 $f(x) = x^3 + 4x^2 - 7x - 10$. For each of $(x - 2)$, $(x + 1)$, $(x + 5)$ and $(x - 5)$, decide whether it is a factor of $f(x)$. You must show your working. [4 marks]

C2 Solve $2x^3 - 5x^2 - 12x = 0$. [3 marks]

Answer $x = \dots, \dots, \dots$

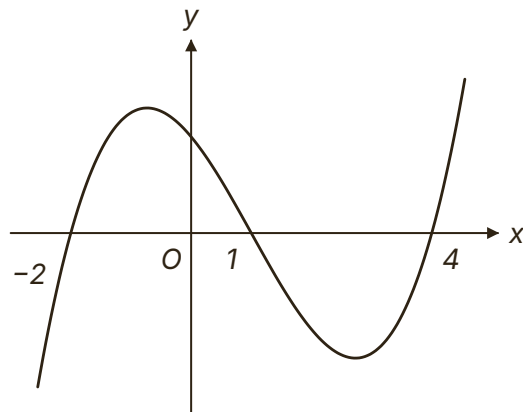
C3 Factorise fully $x^5 - 5x^3 + 4x$. [3 marks]

Answer

C4 $x = -4$ is a root of the equation $x^3 + kx^2 - 7x - 12 = 0$. Work out the value of k and the other two roots. Give the roots in surd form. [5 marks]

Answer $k = \dots$ $x = \dots$

- C5** The sketch shows the graph of $y = x^3 + ax^2 + bx + c$. It crosses the x -axis at $x = -2$, $x = 1$ and $x = 4$. Work out the values of a , b and c . [3 marks]



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- C6** Show that $(x - a)$ is a factor of $x^3 - 2ax^2 + 2a^2x - a^3$. Hence write $x^3 - 2ax^2 + 2a^2x - a^3$ as a product of a linear factor and a quadratic factor. [3 marks]

Answer

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

D Exam-style questions

AQA exam style

D1 $f(x) = 6x^3 + 11x^2 - 19x + 6$

- (a) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. [2 marks]

- (b) Hence solve $f(x) = 0$ [3 marks]

Answer $x = \dots\dots, \dots\dots, \dots\dots$

Total for D1: 5 marks

D2 $f(x) = x^3 - 4x^2 - 3x + 18$. Jo works out $f(3) = 0$ and says, "So $(x + 3)$ is a factor of $f(x)$."

Jo is wrong.

- (a) Show that $(x + 3)$ is not a factor of $f(x)$, and say which factor $f(3) = 0$ does show. [1 mark]

(b) Factorise $f(x)$ fully.

[3 marks]

Answer

Total for D2: 4 marks

D3 $(x + 2)$ and $(x - 5)$ are factors of $x^3 + ax^2 + bx - 40$

(a) Work out the third linear factor.

[2 marks]

Answer (.....)

(b) Work out the values of a and b .

[3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

Total for D3: 5 marks

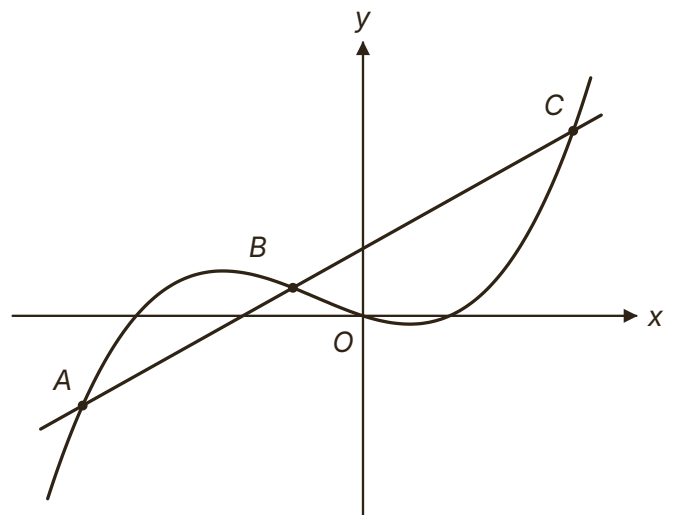
D4 Which of these is a factor of $x^3 + 27$?

[1 mark]

Circle your answer. $(x - 3)$ $(x + 3)$ $(x - 27)$ $(x + 27)$

D5 The curve $y = x^3 + 2x^2 - 4x$ and the line $y = 7x + 12$ meet at the points A , B and C .

(a) Show that the x -coordinates of A , B and C satisfy $x^3 + 2x^2 - 11x - 12 = 0$. [1 mark]



Not drawn accurately

(b) Hence work out the coordinates of A , B and C . You must show your working. [5 marks]

Answer $(\dots, \dots)$, $(\dots, \dots)$ and $(\dots, \dots)$

Total for D5: 6 marks

D6 $(3x - 1)$ is a factor of $3x^3 + kx^2 + 8x - 4$

(a) Work out the value of k . [3 marks]

Answer $k = \dots\dots$

(b) Hence factorise $3x^3 + kx^2 + 8x - 4$ fully. [2 marks]

Answer

Total for D6: 5 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Write $x^6 - 64$ as a product of two linear factors and two quadratic factors. [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** x^6 and 64 are both squares. Start with the difference of two squares.
- Hint 2** $x^6 - 64 = (x^3 - 8)(x^3 + 8)$. Now use the factor theorem on each cubic: try $x = 2$ and $x = -2$.
- Hint 3** $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ and $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$. Neither quadratic has real roots.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can use $f(a) = 0$ to find a factor. ☐ ☐ ☐
- I can factorise a cubic and solve it. ☐ ☐ ☐
- I can find unknown coefficients from factors. ☐ ☐ ☐
- I can show a cubic has only one real root. ☐ ☐ ☐