

The gradient of a curve and differentiation

About 85 minutes No calculator 75 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-the-gradient-of-a-curve-and-differentiation/

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Write $\frac{3}{5x^2}$ in the form ax^n .

A2 Needed today [2 marks]

Work out the gradient of the straight line through $(-2, 7)$ and $(3, -8)$.

Answer

Answer

- A3** Last lesson [2 marks]

Work out the length of the longest diagonal of a cuboid 2 cm by 6 cm by 9 cm.

A4 From chapter 1 [2 marks]

Expand and simplify $(2x - 3)(x^2 + 4x - 1)$.

Answer cm

Answer

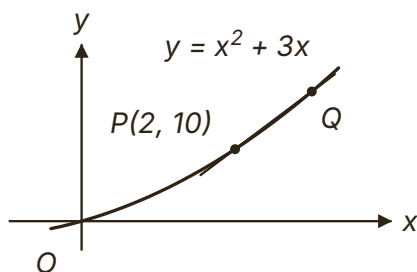
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$P(2, 10)$ and Q are points on $y = x^2 + 3x$. Work out the gradient of PQ when Q has x -coordinate 3, 2.5, 2.1 and 2.01. Hence write down the gradient of the curve at P .

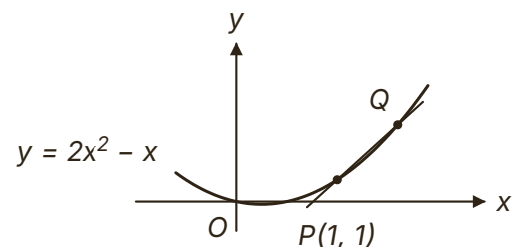


Not drawn accurately

Hint Gradient of PQ = change in $y \div$ change in x . Watch what the gradients get close to.

B1 $P(1, 1)$ and Q are points on the curve $y = 2x^2 - x$. Work out the gradient of PQ when Q has x -coordinate 1.1 and then 1.01. Hence write down the gradient of the curve at P .

[3 marks]



Not drawn accurately

Answer and ; at P :

EXAMPLE 2

Differentiate $y = 3x^4 - \frac{2}{x} + \frac{5}{2x^3} + 7$.

Hint Write every term as a number times a power of x first. The 2 in $\frac{5}{2x^3}$ is not cubed, so that term is $\frac{5}{2}x^{-3}$.

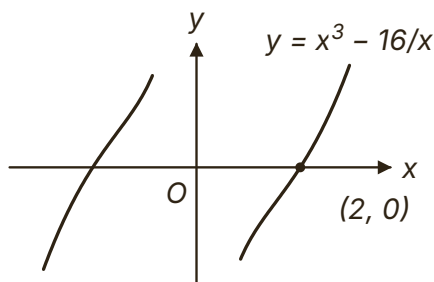
EXAMPLE 3

Work out $\frac{dy}{dx}$ when (a) $y = (2x + 1)(x - 3)^2$ (b) $y = \frac{4x^3 - 6x + 1}{2x^2}$.

Hint You cannot differentiate a product or a fraction term by term. Expand the brackets, or divide each term on the top by $2x^2$, first.

EXAMPLE 4

The curve $y = x^3 - \frac{16}{x}$ crosses the positive x -axis at the point $(2, 0)$. Work out the gradient of the curve at this point.



Not drawn accurately

Hint Work out $\frac{dy}{dx}$, then put $x = 2$ into it. Putting $x = 2$ into y only gives the 0 you were told.

B2 $f(x) = \frac{x^5}{10} + \frac{4}{x^2} - \frac{1}{6x^3} - 9x + 11$. [4 marks]
Work out $f'(x)$.

Answer $f'(x) = \dots\dots$

B3 Work out $\frac{dy}{dx}$ when (a) [5 marks]
 $y = x(3x - 2)(x + 4)$ (b)
 $y = \frac{5x^4 + 3x^2 - 8}{x^3}$.

Answer (a) $\dots\dots$ (b) $\dots\dots$

B4 $y = \frac{x^5 + 24}{4x^2}$. Work out the rate [4 marks]
of change of y with respect to
 x when $x = -2$.

Answer $\dots\dots$

EXAMPLE 5

Work out the coordinates of the points on the curve $y = x^3 - 3x^2 - 7x + 4$ where the gradient is 2.

Hint Put $\frac{dy}{dx} = 2$ and solve. Then use the equation of the curve to find each y .

B5 Work out the exact [4 marks]

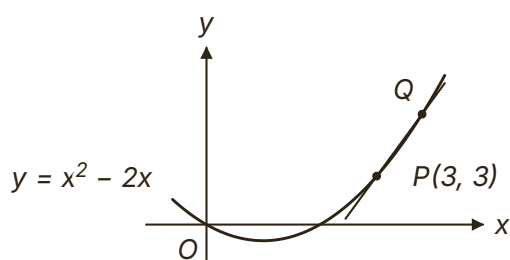
coordinates of the points on the curve $y = 3x + \frac{12}{x}$ where the gradient is -1 .

Answer (.....,) and (.....,)

c Practice

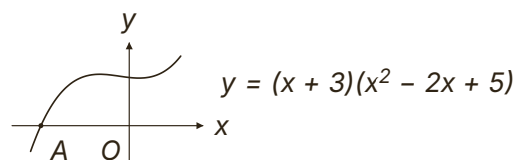
Level 2

C1 $P(3, 3)$ is a point on the curve $y = x^2 - 2x$. Q is the point on the curve with x -coordinate $3 + h$. Show that the gradient of the chord PQ is $4 + h$. Hence write down the gradient of the curve at P . [3 marks]



Not drawn accurately

C2 The curve $y = (x + 3)(x^2 - 2x + 5)$ crosses the x -axis at one point, A . Work out the gradient of the curve at A . [4 marks]



Not drawn accurately

Answer

Answer gradient at P :

- C3** Work out the values of x at which the curves $y = x^3 + x$ and $y = 2x^2 + 5x - 1$ have the same gradient. [3 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

- C4** A rectangle has width x cm and area 36 cm^2 . Its perimeter is P cm. Show that $P = 2x + \frac{72}{x}$. Work out the rate of change of P with respect to x when $x = 4$. [4 marks]

x cm

area 36 cm^2

Not drawn accurately

Answer $\dots\dots$

D Exam-style questions

AQA exam style

- D1** $y = \frac{2}{x^3}$. Which of these is $\frac{dy}{dx}$? Circle your answer. [1 mark]
- $-\frac{6}{x^4}$ $\frac{6}{x^4}$ $-\frac{6}{x^2}$ $\frac{2}{3x^2}$

- D2** Mia is working out the gradient of the curve $y = (x^2 + 1)(3x - 5)$. She writes: " $\frac{dy}{dx} = 2x \times 3 = 6x$." [4 marks]

Explain Mia's mistake. Work out the gradient of the curve at the point where $x = 2$.

Answer gradient = $\dots\dots$

- D3** A curve has equation $y = ax^3 + bx^2$, where a and b are constants. The gradient of the curve is 4 at the point where $x = 1$, and 20 at the point where $x = 2$. [2 marks]
- (a) Show that $3a + 2b = 4$.

(b) Work out the values of a and b .

[3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

(c) Hence work out the coordinates of the other point on the curve where the gradient is 4.

[3 marks]

Answer $(\dots\dots, \dots\dots)$

Total for D3: 8 marks

D4 Water drains out of a tank. The volume of water in the tank, V litres, t minutes after it starts to drain is $V = 2t^3 - 30t^2 + 1200$ for $0 \leq t \leq 10$.

(a) Work out $\frac{dV}{dt}$.

[2 marks]

Answer $\frac{dV}{dt} = \dots\dots$

(b) Work out the rate of change of the volume when $t = 4$. Is the volume increasing or decreasing at this time?

[2 marks]

Answer $\dots\dots$ litres per minute, $\dots\dots$

(c) Show that there is exactly one time at which the volume is decreasing at 150 litres per minute. Work out this time.

[3 marks]

Answer $t = \dots\dots$

Total for D4: 7 marks

D5 A curve has equation $y = \frac{x^3 + 8}{2x}$.

(a) Work out $\frac{dy}{dx}$.

[3 marks]

Answer $\frac{dy}{dx} = \dots\dots$

- (b) Show that there is only one point on the curve where the gradient is 1, and work out its coordinates. You must show your working.

[6 marks]

Answer (.....,)

Total for D5: 9 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Prove that, for every quadratic curve $y = ax^2 + bx + c$, the chord joining the points where $x = p$ and $x = q$ is parallel to the tangent at the point where $x = \frac{p+q}{2}$. [4 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Two lines are parallel when their gradients are equal. Work out the gradient of the chord and the gradient of the curve at $x = \frac{p+q}{2}$, and compare them.
- Hint 2** The gradient of the chord is $\frac{(aq^2 + bq + c) - (ap^2 + bp + c)}{q - p} = \frac{a(q^2 - p^2) + b(q - p)}{q - p}$.
- Hint 3** $q^2 - p^2 = (q - p)(q + p)$, so the chord's gradient is $a(p + q) + b$. $\frac{dy}{dx} = 2ax + b$, which at $x = \frac{p+q}{2}$ is $a(p + q) + b$. The gradients are equal, so the lines are parallel.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find the gradient of a curve at a point from the gradients of chords. ☐ ☐ ☐
- I can differentiate powers of x , including negative powers. ☐ ☐ ☐
- I can expand or divide before differentiating. ☐ ☐ ☐
- I can use $\frac{dy}{dx}$ to find gradients, rates of change and points with a given gradient. ☐ ☐ ☐