

The product rule for counting

About 75 minutes Calculator allowed 65 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-the-product-rule-for-counting/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier lessons.

- A1** Needed today [2 marks] **A2** Needed today [2 marks]
Using each of the cards 2, 5 and 8 once, list every three-digit number that can be made. How many are there?

Answer
- A menu has 4 starters, 6 main courses and 3 desserts. A meal is one of each. How many different meals are there?

Answer

- A3** Last lesson [2 marks] **A4** From chapter 1 [2 marks]
Write $\frac{12}{\sqrt{7}-1}$ in the form $a + b\sqrt{7}$, where a and b are integers.

Answer
- Work out the coefficient of x^2 in the expansion of $(1 + 3x)^4$.

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A club has 9 members. A chair, a secretary and a treasurer are chosen from them. No one can hold two of these jobs. In how many ways can the three jobs be filled?

Hint Fill the jobs one at a time. After the chair is chosen there is one person fewer for the secretary.

- B1** [2 marks]
Eleven horses run in a race. There are no ties. In how many different ways can the first, second and third places be filled?

Answer

EXAMPLE 2

A password is two letters followed by three digits, such as KT407. Any of the 26 letters and any of the 10 digits can be used, and they may repeat. How many passwords are possible?

Hint Repeats are allowed, so every letter place has 26 choices and every digit place has 10.

B2 A code is one letter followed by three digits. The letter is not I or O. The three digits are all different. How many codes are possible? [2 marks]

Answer

EXAMPLE 3

Here are six cards. Each card is used at most once. How many even four-digit numbers can be made?



Hint Fill the place with the condition first. An even number ends in 2 or 4.

B3 The digits 1 to 8 are each used at most once. How many four-digit numbers are multiples of 5? [2 marks]

Answer

EXAMPLE 4

The digits 2, 3, 5, 6, 8 and 9 are each used at most once. How many odd four-digit numbers less than 6000 can be made?

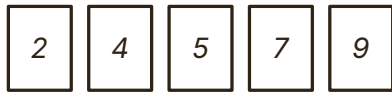
Hint The first digit is 2, 3 or 5 and the last is 3, 5 or 9. Take each first digit in turn, because it changes how many odd digits are left for the end.

B4 The digits 0, 1, 2, 4, 5 and 7 are each used at most once. How many even three-digit numbers can be made? [3 marks]

Answer

EXAMPLE 5

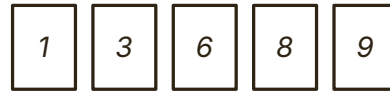
Here are five cards. Using one, two or three of the cards, how many numbers less than 500 can be made?



Hint Count the one-digit, two-digit and three-digit numbers separately. Only the three-digit ones can be 500 or more.

B5 Here are five cards. Using [3 marks]

three or four of the cards, how many numbers greater than 800 can be made?



Answer

c Practice

Level 2

C1 The five letters of the word FOCUS are arranged in a line.

- (a) How many different [1 mark]
arrangements are there?

Answer

- (b) How many of the [1 mark]
arrangements begin with F?

Answer

- (c) How many of the [2 marks]
arrangements begin with a
vowel and end with a
vowel?

Answer

Total for C1: 4 marks

C2 A padlock has four wheels. Each wheel shows a digit from 0 to 9. A code is the four digits in order.

- (a) How many codes are [1 mark]
possible?

Answer

- (b) How many codes have four [2 marks]
different digits?

Answer

- (c) How many codes have at [1 mark]
least two digits the same?

Answer

Total for C2: 4 marks

C3 A meal deal is one of 5 sandwiches, one of 4 snacks and one of 6 drinks.

- (a) How many different meal deals are there? [1 mark]

Answer

- (b) Ben will not have either of the 2 fizzy drinks or the 1 snack with nuts. How many meal deals could he choose? [2 marks]

Answer

Total for C3: 3 marks

C4 Seven people, including Amy and Ben, stand in a line.

- (a) Amy must stand at the left-hand end. How many arrangements are possible? [1 mark]

Answer

- (b) Amy and Ben must stand at the two ends, either way round. How many arrangements are possible? [2 marks]

Answer

Total for C4: 3 marks

C5 How many three-digit numbers have three different digits? [2 marks]

Answer

C6 Digits may be repeated. How many four-digit numbers are even? [2 marks]

Answer

D Exam-style questions

AQA exam style

D1 Lena says, "Four of the five cards are even, so $\frac{4}{5}$ of the 120 four-digit numbers are even. That is 96 even numbers." [3 marks]

Lena makes four-digit numbers from these cards, using each card at most once. Explain why Lena is wrong, and work out the number of even four-digit numbers.

2	4	5	8	9
---	---	---	---	---

Answer

D2 Tom's date of birth is 27/03/2008. He makes a 4-digit code using digits from his date of birth. The code must not start with 0 and must have all different digits.

- (a) How many codes can he make? [2 marks]

Answer

(b) How many of these codes are odd numbers?

[3 marks]

Answer

Total for D2: 5 marks

D3 Integers are made using some or all of the digits 2, 3, 5, 7, 8 and 9. No digit is repeated. Each integer is greater than 70 000. How many integers can be made? [4 marks]

Answer

D4 A code is two letters followed by three digits. The letters are chosen from the 24 letters other than I and O, and they may be the same. The three digits are all different, and the code must not end in 0.

(a) Work out the number of possible codes. [3 marks]

Answer

(b) The rule changes so that the digits may repeat. The code still must not end in 0. How many more codes are there now? [2 marks]

Answer

Total for D4: 5 marks

D5 Four girls and three boys sit in a row of seven seats. Boys and girls must sit in alternate seats. How many arrangements are possible? [3 marks]

Answer

D6 A group has 10 people.

- (a) A captain, a vice-captain and a secretary are chosen from the group. Each person has at most one job. In how many ways can this be done? [1 mark]

Answer

- (b) Instead, a committee of 3 people is chosen from the group, with no jobs. Using your answer to part (a), work out the number of different committees. [2 marks]

Answer

Total for D6: 3 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** In the number 1358 the digits are all different and increase from left to right. How many four-digit numbers have digits that all increase from left to right? [4 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Choose four different digits first. How many ways are there to put them in increasing order?
- Hint 2** The first digit cannot be 0, and 0 cannot be anywhere later either, so the four digits come from 1 to 9.
- Hint 3** Ordered choices of four digits from nine:
 $9 \times 8 \times 7 \times 6 = 3024$. Each set of four digits appears $4 \times 3 \times 2 \times 1 = 24$ times, and only one of those is increasing. So $3024 \div 24 = 126$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can count arrangements by multiplying the choices for each place. ☐ ☐ ☐
- I can fill a place with a condition first, such as the first or last digit. ☐ ☐ ☐
- I can split a count into cases and add them. ☐ ☐ ☐
- I can count codes and numbers with or without repeats. ☐ ☐ ☐