



The sine rule

About 90 minutes Calculator allowed 66 marks

Date _____

A Do Now

Answer these before the lesson starts.

- A1** GCSE Higher [2 marks]

In triangle ABC , angle $B = 90^\circ$, $AC = 17$ cm and angle $A = 29^\circ$.
Work out BC to 3 significant figures.

A2 Needed today [2 marks]

Solve $\sin \theta = 0.43$ for $0^\circ < \theta < 180^\circ$. Give your answers to 1 decimal place.

Answer $\theta = \dots\dots$ or $\theta = \dots\dots$

- A3** Last lesson [2 marks]

A triangle has sides of 11 cm and 13 cm with an angle of 47° between them. Work out its area to 3 significant figures.

A4 From chapter 6 [2 marks]

Write down the exact values of $\sin 60^\circ$ and $\sin 135^\circ$.

Answer $\sin 60^\circ = \dots\dots$ $\sin 135^\circ = \dots\dots$

Answer $\dots\dots \text{ cm}^2$

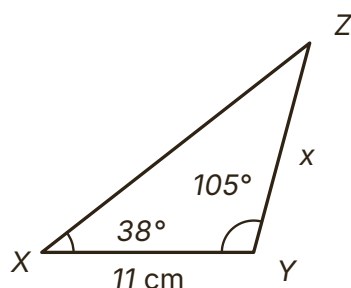
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Give lengths and angles to 3 significant figures unless the question says otherwise.

EXAMPLE 1

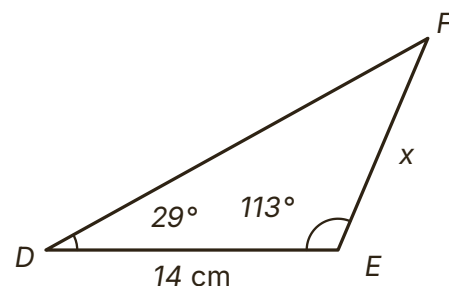
Work out the length x .



Not drawn accurately

Hint Pair each side with the angle opposite it:
 $\frac{a}{\sin A} = \frac{b}{\sin B}$. The angle opposite the 11 cm side is not given, so find it from the angle sum first.

- B1** Work out the length x . [3 marks]

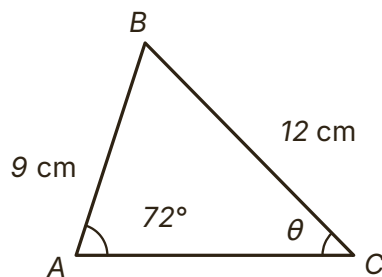


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Answer $x = \dots\dots$ cm

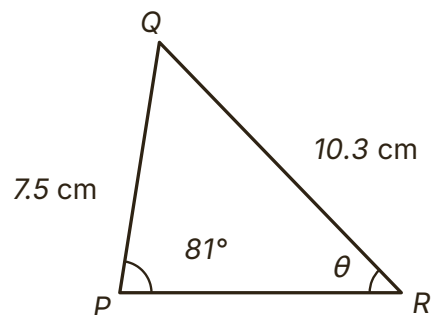
EXAMPLE 2

Work out the size of angle θ .



Not drawn accurately

Hint To find an angle, use the form $\frac{\sin A}{a} = \frac{\sin C}{c}$, with the angle on top.

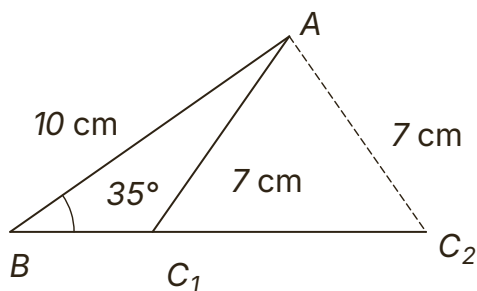
B2 Work out the size of angle θ . [3 marks]

Not drawn accurately

Answer $\theta = \dots\dots^\circ$

EXAMPLE 3

In triangle ABC , angle $B = 35^\circ$, $AB = 10$ cm and $AC = 7$ cm. Work out the two possible sizes of angle C .



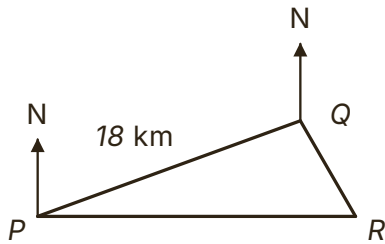
Hint The diagram shows why there are two triangles: the 7 cm side can swing to meet the base at C_1 or at C_2 .

B3 In triangle PQR , angle $Q = 28^\circ$, $PQ = 11$ cm and $PR = 6.5$ cm. Work out the two possible sizes of angle R . [3 marks]

Answer angle $R = \dots\dots^\circ$ or $\dots\dots^\circ$

EXAMPLE 4

A ship sails 18 km from port P on a bearing of 070° to Q . It then sails on a bearing of 150° to R , which is due east of P . Work out the distance PR .

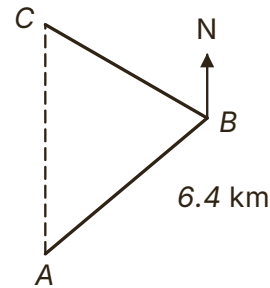


Not drawn accurately

Hint Angle $QPR = 90^\circ - 70^\circ$. At Q , the bearing back to P is 250° , so angle $PQR = 250^\circ - 150^\circ$.

B4 [4 marks]

Jo walks 6.4 km from A on a bearing of 050° to B . She then walks on a bearing of 300° until she reaches C , which is due north of A . How far is she from A ?



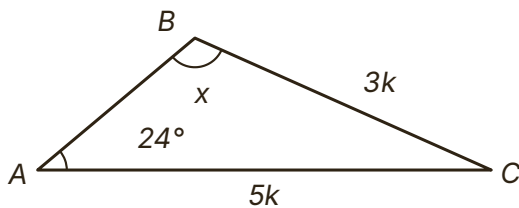
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Answer km

c Practice

Level 2

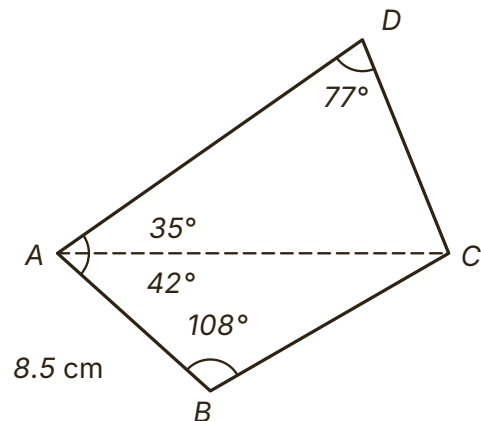
- C1** In the triangle, k is a positive number. Use the sine rule to work out the size of the obtuse angle x . [3 marks]



Not drawn accurately

Answer $x = \dots\dots^\circ$

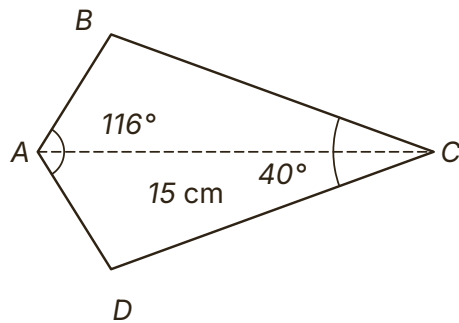
- C2** $ABCD$ is a quadrilateral. Work out the length of CD . [4 marks]



Not drawn accurately

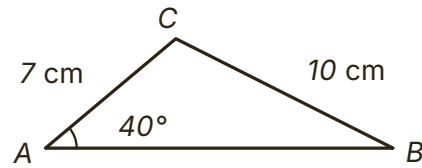
Answer $CD = \dots\dots$ cm

- C3** $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonal AC is 15 cm. Angle $BAD = 116^\circ$ and angle $BCD = 40^\circ$. Work out the perimeter of the kite. [3 marks]



Not drawn accurately

- C4** In triangle ABC , angle $A = 40^\circ$, $BC = 10$ cm and $AC = 7$ cm. Explain why angle B has only one possible size, and work it out. [3 marks]



Not drawn accurately

Answer angle $B = \dots\dots^\circ$

Answer $\dots\dots$ cm

D Exam-style questions

AQA exam style

- D1** In triangle PQR , angle $P = 42^\circ$, $QR = 8$ cm and $PR = 11$ cm. Ravi says, "Angle Q is 66.9° . There is only one triangle with these measurements."

(a) Explain why Ravi is wrong. [2 marks]

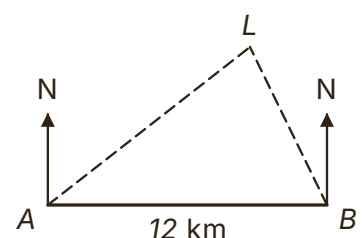
(b) Work out the difference between the two possible lengths of PQ . [3 marks]

Answer $\dots\dots$ cm

Total for D1: 5 marks

- D2** A boat is at A . A lighthouse L is on a bearing of 052° from A . The boat sails 12 km due east to B . From B , the lighthouse is on a bearing of 334° .

(a) Show that angle $ALB = 78^\circ$. [2 marks]



Not drawn accurately

(b) Work out the distance BL .

[2 marks]

Answer $BL = \dots\dots$ km

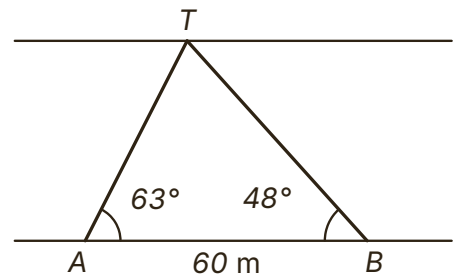
(c) The boat sails back towards A . Work out the shortest distance between the boat and the lighthouse on this journey.

[2 marks]

Answer $\dots\dots$ km

Total for D2: 6 marks

- D3** Two posts A and B stand 60 m apart on one bank of a river. A tree T is on the other bank. The banks are parallel. Angle $TAB = 63^\circ$ and angle $TBA = 48^\circ$. Work out the width of the river. You must show your working. [5 marks]



Answer $\dots\dots$ m

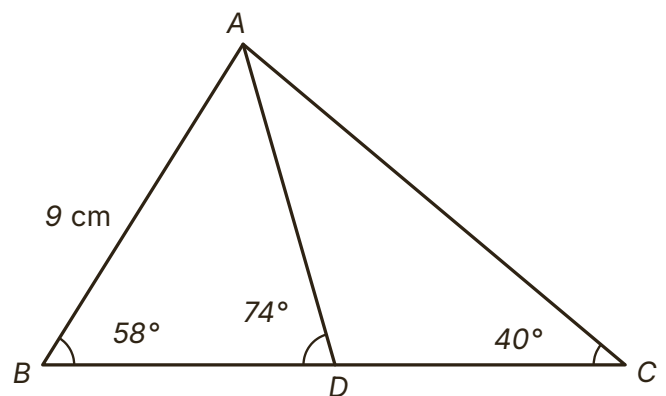
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- D4** D is a point on the side BC of triangle ABC . $AB = 9$ cm, angle $ABD = 58^\circ$, angle $ADB = 74^\circ$ and angle $ACD = 40^\circ$.

(a) Work out the length of AD . [2 marks]

Answer $AD = \dots\dots$ cm

(b) Work out the length of AC . [2 marks]



Not drawn accurately

Answer $AC = \dots\dots$ cm

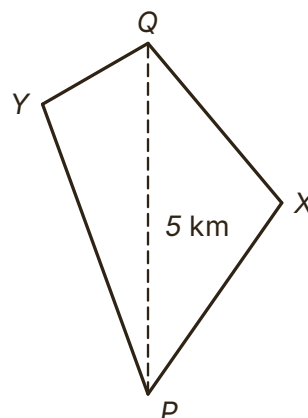
(c) Work out the area of triangle ADC .

[2 marks]

Answer $\dots\dots$ cm²

Total for D4: 6 marks

- D5** Q is 5 km due north of P . Route 1 goes from P on a bearing of 035° to X , then on a bearing of 320° to Q . Route 2 goes from P on a bearing of 340° to Y , then on a bearing of 060° to Q . Which route is shorter, and by how much? Give your answer to 3 decimal places. [6 marks]



Answer Route is shorter by km

Not drawn accurately

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** In triangle ABC , angle $A = 35^\circ$ and $BC = 6$ cm. The length of AC is b cm. Find the set of values of b for which there are two different triangles ABC . Give the end values to 3 significant figures where needed. [4 marks]

Answer $< b <$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Try $b = 5$, $b = 8$ and $b = 12$. For each, work out $\sin B$ and decide how many triangles there are.
- Hint 2** $\sin B = \frac{b \sin 35^\circ}{6}$. Two angles B need $\sin B < 1$, and the obtuse one must fit with $A = 35^\circ$.
- Hint 3** $\sin B < 1$ gives $b < \frac{6}{\sin 35^\circ} = 10.46$. The obtuse $B = 180^\circ - B_1$ fits only if $180^\circ - B_1 < 145^\circ$, that is $B_1 > 35^\circ$, so $\sin B > \sin 35^\circ$ and $b > 6$. So $6 < b < 10.5$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can use the sine rule to find a side when I know two angles and a side. ☐ ☐ ☐
- I can use the sine rule to find an angle when I know two sides and an angle not between them. ☐ ☐ ☐
- I can tell when there are two possible triangles and find both answers. ☐ ☐ ☐
- I can use the sine rule in bearings and multi-step problems. ☐ ☐ ☐