

Transformations and the unit square

About 80 minutes No calculator 62 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-transformations-and-the-unit-square/

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** Last lesson [2 marks]
- Write down the image of the point $(5, -2)$ after (a) a rotation of 90° anticlockwise about O and (b) a reflection in the line $y = -x$.
- Answer

Answer

- A3** Last lesson [2 marks] **A4** From chapter 8 [2 marks]
- Given that $\begin{pmatrix} 3 & k \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix} = 2\mathbf{I}$, work out the value of k .
- Work out the x -coordinates of the stationary points of $y = x^3 - 6x^2 + 9x$.

Answer $k = \dots\dots$

Answer $x = \dots\dots$ and $x = \dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the image of the point $(-4, 3)$ under the transformation matrix $\begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix}$.

Hint Write the point as a column vector and multiply: the matrix goes first.

- B1** [2 marks] Work out the image of the point $(6, -2)$ under the transformation matrix $\begin{pmatrix} -3 & 1 \\ 4 & -2 \end{pmatrix}$.

Answer $(\dots\dots, \dots\dots)$

EXAMPLE 2

The matrix $\begin{pmatrix} k & 3 \\ 2 & -1 \end{pmatrix}$ maps the point $(4, -2)$ to the point $(14, 10)$. Work out the value of k .

Hint Multiply the matrix by the column vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and set the top entry equal to 14.

B2 The matrix $\begin{pmatrix} 2 & p \\ q & -3 \end{pmatrix}$ maps the point $(3, 4)$ to the point $(-2, 15)$. Work out the values of p and q . [3 marks]

Answer $p = \dots\dots$ $q = \dots\dots$

EXAMPLE 3

The matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ maps the point P to the point $(9, 13)$. Work out the coordinates of P .

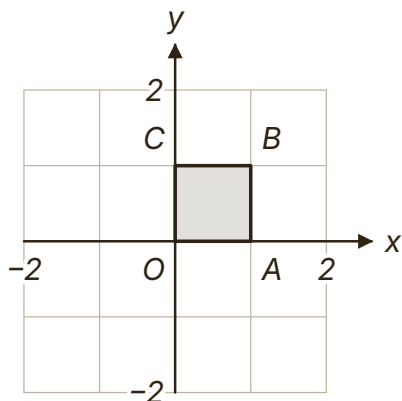
Hint Call P the point (x, y) . Multiplying gives two equations in x and y . Solve them.

B3 The matrix $\begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}$ maps the point Q to the point $(-11, -2)$. Work out the coordinates of Q . [3 marks]

Answer $(\dots\dots, \dots\dots)$

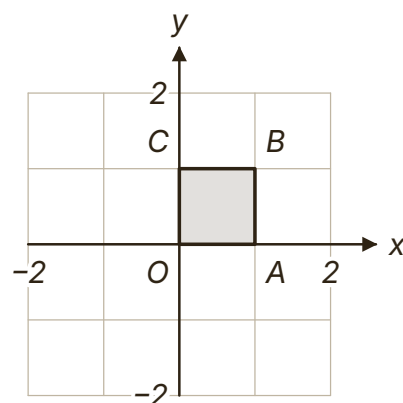
EXAMPLE 4

Work out the matrix for a rotation of 90° anticlockwise about the origin. Show the image of the unit square $OABC$ on the grid, labelling each vertex.



Hint Find where $(1, 0)$ and $(0, 1)$ go. Their images, written as columns side by side, are the matrix.

B4 Work out the matrix for a reflection in the line $y = -x$. On the grid, draw the image of the unit square $OABC$ under this reflection and label A' , B' and C' . [3 marks]



Answer matrix = $\dots\dots\dots$

EXAMPLE 5

Describe fully the single transformation given by the matrix $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$. The unit square is transformed by this matrix. Work out the area of the image.

Hint Find the images of $(1, 0)$ and $(0, 1)$. Then the lengths of the sides of the image.

- B5** The unit square $OABC$ is transformed by the matrix $\begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}$ to $OA'B'C'$. The area of $OA'B'C'$ is 50. Work out the two possible values of h . Give your answers in surd form. [3 marks]

Answer $h = \dots\dots$ or $h = \dots\dots$

c Practice

Level 2

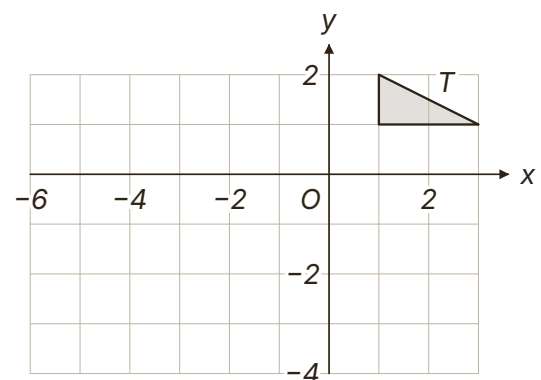
- C1** Describe fully the single transformation given by the matrix $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$. [2 marks]
- C2** The matrix $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ maps the point $(a, 4)$ to the point $(b, 25)$. Work out the values of a and b . [3 marks]

Answer

Answer $a = \dots\dots$ $b = \dots\dots$

- C3** A transformation maps $(1, 0)$ to $(2, 5)$ and $(0, 1)$ to $(-1, 3)$. Write down its matrix. Hence work out the image of the point $(4, -2)$. [3 marks]
- C4** Triangle T has vertices $(1, 1)$, $(3, 1)$, $(1, 2)$. It is transformed by $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$. Work out the vertices of the image of T , and its area. [4 marks]

Answer matrix = image = (.....,)



Answer vertices area =

- C5** A point P is reflected in the line $y = x$. Its image is $(-4, 9)$. Write down the matrix for the reflection, and use it to work out the coordinates of P . [2 marks]

Answer $P = (\dots, \dots)$

D Exam-style questions

AQA exam style

- D1** Which matrix represents a reflection in the line $y = x$? Circle your answer. [1 mark]

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

- D2** Ella says, "The matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of 90° clockwise about the origin." [3 marks]

Show that Ella is wrong. Then write down the matrix for a rotation of 90° clockwise about the origin.

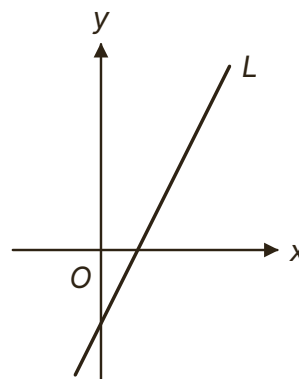
Answer matrix =

- D3** The matrix $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ maps the point (p, q) to the point $(p + 8, 2q)$. Work out the values of p and q . You must show your working. [4 marks]

Answer $p = \dots$ $q = \dots$

- D4** The line L has equation $y = 2x - 1$. Every point of L is transformed by the matrix $\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$. [4 marks]

Show that every image point lies on the line $y = 5x - 7$.



Not drawn accurately

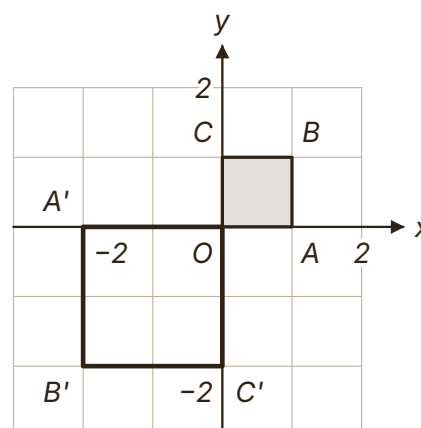
- D5** The diagram shows the unit square $OABC$ and its image $OA'B'C'$ under a transformation T .

- (a) Write down the matrix for T . [2 marks]

Answer

- (b) Describe fully the single transformation T . [2 marks]

Answer



- (c) T maps the point Q to the point $(8, -6)$. Work out the coordinates of Q . [2 marks]

Answer (.....,)

Total for D5: 6 marks

- D6** $\mathbf{M} = \begin{pmatrix} 2 & k \\ k & 2 \end{pmatrix}$, where k is a constant. $\mathbf{M}$ maps the point $(3, -1)$ to a point on the line $x + y = 10$. Work out the value of k , and the coordinates of the image point. [3 marks]

Answer $k = \dots\dots$ (.....,)

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The unit square $OABC$ is transformed by the matrix $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$ to $OA'B'C'$. [5 marks]
Show that $OA'B'C'$ is a square, and work out its area.

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Find A' , B' and C' by multiplying. Then look at the lengths of the sides and the angle at O .
- Hint 2** $A' = (3, 4)$, $B' = (-1, 7)$ and $C' = (-4, 3)$. The length OA' is $\sqrt{3^2 + 4^2}$.
- Hint 3** $OA' = A'B' = B'C' = C'O = 5$. The gradient of OA' is $\frac{4}{3}$ and of OC' is $-\frac{3}{4}$; their product is -1 , so the angle at O is a right angle. A rhombus with a right angle is a square, of area $5^2 = 25$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can work out the image of a point under a matrix, and the point that maps to a given image. ☐ ☐ ☐
- I can write down the matrix for a reflection, a rotation about the origin or an enlargement centre the origin. ☐ ☐ ☐
- I can draw the image of the unit square and describe fully the transformation a matrix represents. ☐ ☐ ☐
- I can find an unknown in a matrix or a point from a given image or area. ☐ ☐ ☐