

Trigonometric functions and their graphs

About 75 minutes No calculator 63 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-trigonometric-functions-and-their-graphs/

Date _____

A Do Now

Four questions to start. Two of them use what you need today.

A1 Needed today [2 marks] **A2** Last lesson [2 marks]

Work out the exact value of
 $\sin 60^\circ \times \tan 30^\circ + \cos 45^\circ \times \sin 45^\circ$.

Answer

A ladder 8 m long leans against a vertical wall. It makes an angle of 60° with the horizontal ground. Work out exactly how high up the wall the ladder reaches.

Answer

A3 From chapter 5 [2 marks] **A4** From chapter 4 [2 marks]

A circle has equation
 $x^2 + y^2 - 6x + 4y = 12$. Work out the centre and the radius of the circle.

$(x - 2)$ is a factor of $x^3 + kx - 6$.
Work out the value of k .

Answer $k = \dots\dots$

Answer centre $(\dots, \dots)$ radius ...

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the exact value of
 $\cos 30^\circ + \cos 150^\circ + \sin 240^\circ$. Give your answer in its simplest form.

Hint Write each ratio as $\pm$ a ratio of an acute angle:
 $\cos 150^\circ = -\cos 30^\circ$ and $\sin 240^\circ = -\sin 60^\circ$.

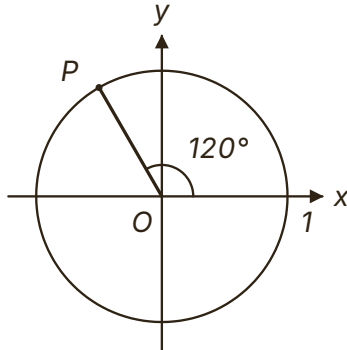
B1 Work out the exact value of [3 marks]

$\sin 45^\circ \times \cos 135^\circ + \tan 240^\circ$.
Give your answer in its simplest form.

Answer

EXAMPLE 2

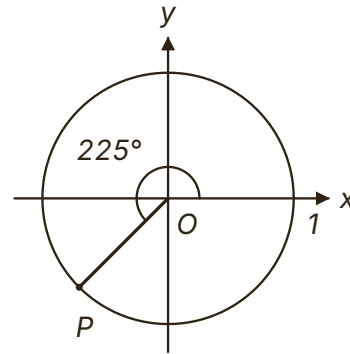
The point P lies on the circle $x^2 + y^2 = 1$, and OP makes an angle of 120° with the positive x -axis. Write down the exact coordinates of P . Hence write down the exact values of $\sin 120^\circ$, $\cos 120^\circ$ and $\tan 120^\circ$.



Hint OP makes 60° with the negative x -axis. P is $(\cos 120^\circ, \sin 120^\circ)$, and $\tan 120^\circ = \frac{y}{x}$.

B2 [3 marks]

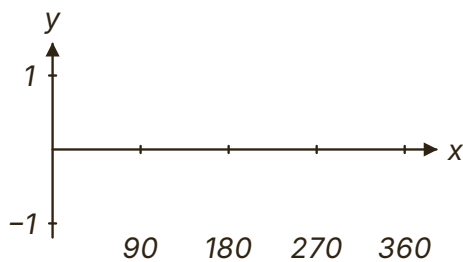
The point P lies on the circle $x^2 + y^2 = 1$, and OP makes an angle of 225° with the positive x -axis. Write down the exact coordinates of P . Hence write down the exact values of $\sin 225^\circ$, $\cos 225^\circ$ and $\tan 225^\circ$.



Answer

EXAMPLE 3

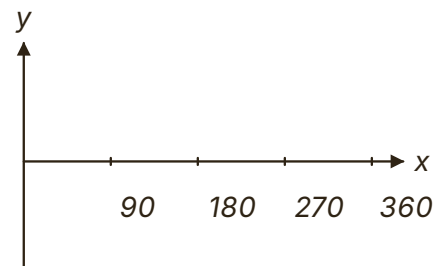
Sketch the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ on the axes. Write down the coordinates of the points where it meets the axes, and of its minimum point.



Hint Start at $(0, 1)$. The cosine graph is the sine graph moved 90° to the left.

B3 [3 marks]

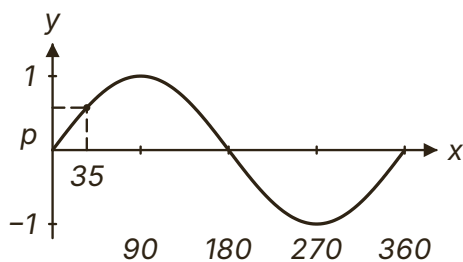
Sketch the graph of $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$ on the axes. Write down the equations of its asymptotes, and the coordinates of the points where it crosses the x -axis.



Answer

EXAMPLE 4

Here is the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$. You are given that $\sin 35^\circ = p$. Work out the two values of x , for $0^\circ \leq x \leq 360^\circ$, for which $\sin x = -p$.



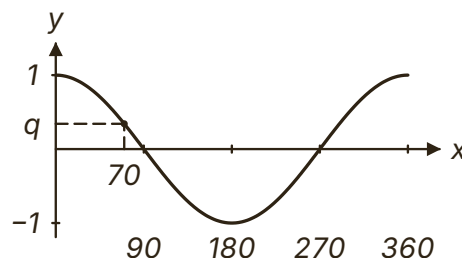
Hint The curve between 180° and 360° is the curve between 0° and 180° turned upside down.

EXAMPLE 5

Work out the exact values of $\sin 480^\circ$, $\cos(-120^\circ)$ and $\tan 570^\circ$.

Hint Add or take away 360° until the angle is between 0° and 360° ; the tangent also repeats every 180° .

B4 Here is the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$. You are given that $\cos 70^\circ = q$. Work out the two values of x , for $0^\circ \leq x \leq 360^\circ$, for which $\cos x = -q$.



Answer $x = \dots\dots$ and $x = \dots\dots$

B5 Work out the exact values of $\cos 390^\circ$, $\sin(-150^\circ)$ and $\tan(-225^\circ)$.

Answer

c Practice

Level 2

C1 Write true or false for each statement. Give a reason for each one. [4 marks]

(i) $\sin 50^\circ = \sin 130^\circ$

(ii) $\cos 40^\circ = \cos 140^\circ$

(iii) $\tan 20^\circ = \tan 160^\circ$

(iv) $\sin 250^\circ = \sin 290^\circ$

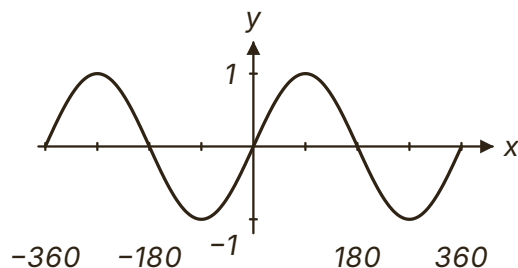
C2 The angle θ is between 180° and 270° . For each of $\sin \theta$, $\cos \theta$ and $\tan \theta$, write down whether it is positive or negative. [2 marks]

Answer

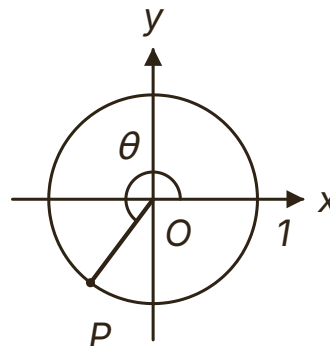
Answer (i) ... (ii) ... (iii) ... (iv) ...

- C3** Here is the graph of $y = \sin x$ for $-360^\circ \leq x \leq 360^\circ$. Use it to find how many solutions each equation has for $-360^\circ \leq x \leq 360^\circ$.

(i) $\sin x = -0.7$ (ii) $\sin x = 0$



- C4** The point $P(-0.6, -0.8)$ lies on the circle $x^2 + y^2 = 1$. OP makes the angle θ with the positive x -axis. Write down the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$.



Answer (i) ... (ii) ...

Answer

- C5** Explain why $\tan 90^\circ$ has no value. Write down the next angle after 90° for which $\tan x$ has no value.

- C6** Describe fully the single transformation that maps the graph of $y = \sin x$ onto the graph of $y = \cos x$.

Answer next angle

Answer

D Exam-style questions

AQA exam style

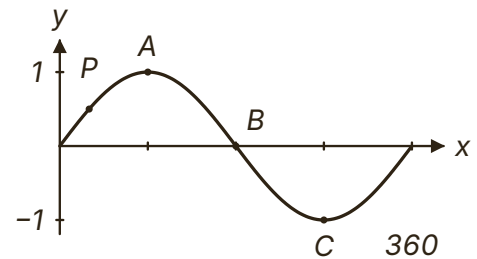
- D1** Ravi says, " $\sin 200^\circ = \sin 20^\circ$, because the sine graph repeats itself every 180° ." [2 marks]

Explain why Ravi is wrong. Write $\sin 200^\circ$ in terms of $\sin 20^\circ$.

Answer $\sin 200^\circ = \dots\dots$

D2 Here is a sketch of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$. A is the maximum point, C is the minimum point and B is where the curve crosses the x -axis between them. The point $P \left(30, \frac{1}{2} \right)$ lies on the curve.

- (a) Write down the coordinates of A , B and C . [2 marks]



Answer $A(\dots, \dots)$ $B(\dots, \dots)$ $C(\dots, \dots)$

- (b) The points Q and R also lie on the curve. They both have y -coordinate $-\frac{1}{2}$. Work out the x -coordinates of Q and R . [2 marks]

Answer $x = \dots\dots$ and $x = \dots\dots$

- (c) Hence write down the two solutions of $\sin x = \frac{1}{2}$ for $360^\circ \leq x \leq 720^\circ$. [2 marks]

Answer $x = \dots\dots$ and $x = \dots\dots$

Total for D2: 6 marks

D3 Show that $\tan^2 120^\circ - 4 \sin 210^\circ \cos 300^\circ = 4$. You must show your working. [4 marks]

D4 Which of these is equal to $\cos 50^\circ$? Circle your answer. [1 mark]

$\cos 130^\circ$ $\cos 230^\circ$ $\cos 310^\circ$ $-\cos 310^\circ$

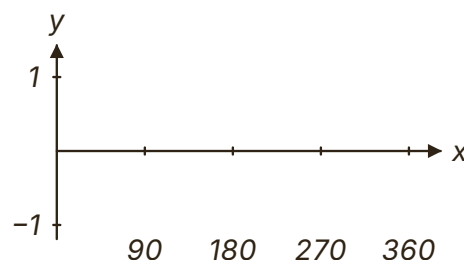
Answer

D5 θ is an obtuse angle and $\sin \theta = \frac{5}{13}$. Work out the exact values of $\cos \theta$ and $\tan \theta$. Do not use a calculator. [3 marks]

Answer $\cos \theta = \dots\dots$ $\tan \theta = \dots\dots$

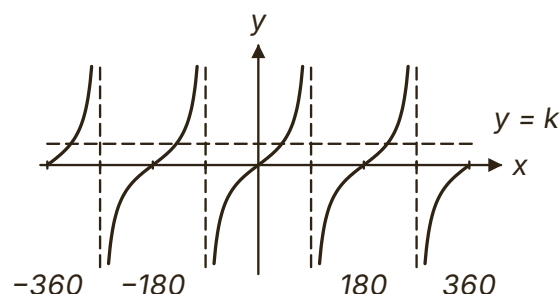
- D6** On the axes, sketch the graphs of $y = \sin x$ and $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$. Hence [4 marks]
find the range of values of x for which $\sin x > \cos x$. You must show your working.

Answer $\dots\dots < x < \dots\dots$



- D7** The graph shows $y = \tan x$ for $-360^\circ \leq x \leq 360^\circ$ and the line $y = k$. The line meets [3 marks]
the curve at the point where $x = 40^\circ$. Work out the other values of x , for
 $-360^\circ \leq x \leq 360^\circ$, where the line meets the curve.

Answer $x = \dots\dots$



E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve $y = a \cos x + b$, where a and b are constants and $a > 0$, has a greatest [4 marks]
value of 7 and a least value of -3 . Work out a and b . Hence find the values of x ,
for $0^\circ \leq x \leq 360^\circ$, for which $y = 4.5$. Do not use a calculator.

Answer $a = \dots$ $b = \dots$ $x = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $\cos x$ goes from -1 to 1 , so y goes from $-a + b$ to $a + b$.
Hint 2 $a + b = 7$ and $-a + b = -3$.
Hint 3 $a = 5$ and $b = 2$, so $5 \cos x + 2 = 4.5$, $\cos x = \frac{1}{2}$, which
gives $x = 60^\circ$ and $x = 360 - 60 = 300^\circ$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the unit circle to find $\sin$,
 $\cos$ and $\tan$ of any angle.



I can sketch the graphs of sine,
cosine and tangent and give their
key points.



I can use the symmetry of the
graphs to find exact values and
related angles.

