



Trigonometric identities

About 80 minutes Calculator allowed 73 marks

Date _____

A Do Now

Four questions to start. Two of them use what you need today.

- A1** Needed today [2 marks]

Solve $3y^2 - 2y - 1 = 0$.

Answer $y = \dots\dots$ or $y = \dots\dots$

A2 Last lesson [2 marks]

Solve $5 \sin x = 2$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place.

Answer $x = \dots\dots$ or $x = \dots\dots$

- A3** From chapter 6 [2 marks]

Do not use a calculator. Work out the exact value of $\cos 30^\circ \times \tan 60^\circ$.

Answer _____

A4 From chapter 2 [2 marks]

Simplify fully $\frac{x^2 - 9}{x^2 - x - 6}$.

Answer _____

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Use the identities $\sin^2 x + \cos^2 x \equiv 1$ and $\tan x \equiv \sin x \div \cos x$.

EXAMPLE 1

Prove that $\frac{\sin^2 x}{1 + \cos x} \equiv 1 - \cos x$.

Hint Start with the harder side. Write $\sin^2 x$ as $1 - \cos^2 x$, then factorise it as a difference of two squares.

- B1** Prove that [3 marks]

$$\frac{\cos^2 x}{1 - \sin x} - \sin x \equiv 1.$$

EXAMPLE 2

Show that $(2 \cos x + \sin x)^2 + (\cos x - 2 \sin x)^2$ simplifies to an integer.

Hint Expand both brackets. The $\sin x \cos x$ terms cancel. Collect the $\sin^2 x$ and $\cos^2 x$ terms.

- B2** Show that [3 marks]

$(3 \sin x - \cos x)^2 + (\sin x + 3 \cos x)^2 - 4$ simplifies to an integer.

Answer

EXAMPLE 3

Write $\sin x \tan x + \cos x$ as a single fraction in terms of $\cos x$ only.

Hint Write $\tan x$ as $\frac{\sin x}{\cos x}$, put everything over $\cos x$, then use $\sin^2 x + \cos^2 x = 1$.

B3 Write $\frac{\cos x}{\tan x} + \sin x$ as a single fraction in terms of $\sin x$ only. [2 marks]

Answer

EXAMPLE 4

Do not use a calculator. Solve $2 \sin^2 x + 3 \cos x - 3 = 0$ for $0^\circ \leq x \leq 360^\circ$.

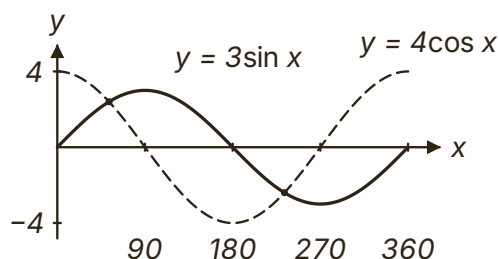
Hint Replace $\sin^2 x$ with $1 - \cos^2 x$. This gives a quadratic in $\cos x$ only. Factorise it.

B4 Solve $6 \cos^2 x + \sin x - 5 = 0$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place where they are not exact. [4 marks]

Answer $x = \dots\dots$

EXAMPLE 5

Solve $3 \sin x = 4 \cos x$ for $0^\circ \leq x \leq 360^\circ$.



Hint Divide both sides by $3 \cos x$. Then $\frac{\sin x}{\cos x}$ is $\tan x$.

B5 Solve $5 \sin x + 2 \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$. [3 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

c Practice

Level 2

C1 Prove that $(\sin x + 2 \cos x)(2 \sin x + \cos x)$ is identical to $2 + 5 \sin x \cos x$. [3 marks]

C2 Write $\frac{\cos^2 x}{1 + \sin x}$ in terms of $\sin x$. Give your answer without a fraction. [2 marks]

Answer

- C3** Solve $3 \sin x = 7 \cos x$ for $-180^\circ \leq x \leq 180^\circ$. [3 marks]
- C4** Do not use a calculator. $\cos x = -\frac{3}{7}$ and $180^\circ < x < 270^\circ$. [4 marks]
Work out the exact value of $\sin x$ and the exact value of $\tan x$.

Answer $x = \dots\dots$ or $x = \dots\dots$

- C5** Solve $4 \sin x = 3 \tan x$ for $0^\circ \leq x \leq 360^\circ$. [4 marks]
- C6** Solve $\cos^2 x + 3 \sin x = 2$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place. [4 marks]

Answer $x = \dots\dots$

Answer $\sin x = \dots\dots$ $\tan x = \dots\dots$

D Exam-style questions

AQA exam style

- D1** Sam says, " $(\sin x + \cos x)^2 \equiv 1$, because $\sin^2 x + \cos^2 x \equiv 1$." [3 marks]
Show that Sam is wrong.

- D2** Show that $\frac{(\sin x + \cos x)^2 - 1}{2 \cos x} - \frac{3 \sin^2 x - 3}{\cos x}$ can be written in the form $a \cos x + b \sin x$, where a and b are integers. [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

- D3** Do not use a calculator. x is an acute angle and $\tan x = 2$. Work out the exact value of $\sin x \cos x$. [3 marks]

Answer $\dots\dots$

D4 Answer both parts.

(a) Show that $3 \sin^2 x - \cos^2 x \equiv 4 \sin^2 x - 1$. [1 mark]

(b) Hence solve $3 \sin^2 x - \cos^2 x = 2 \sin x$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to the nearest degree. [4 marks]

Answer $x = \dots\dots$

Total for D4: 5 marks

D5 Solve $2 \sin^2 x = 3 \sin x \cos x$ for $0^\circ \leq x \leq 360^\circ$. You must show your working. [4 marks]

Answer $x = \dots\dots$

D6 Answer both parts.

(a) Prove that $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [3 marks]

(b) Hence, without a calculator, work out the exact value of $\tan 30^\circ + \frac{1}{\tan 30^\circ}$. Give [2 marks]
your answer in the form $\frac{a\sqrt{3}}{b}$, where a and b are integers.

Answer $\dots\dots$

Total for D6: 5 marks

D7 Which of these is equal to $\frac{1 - \cos^2 x}{\sin x \cos x}$? Circle your answer. [1 mark]
 $\sin x$ $\cos x$ $\tan x$ $1 \div \tan x$

Answer

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Do not use a calculator. Solve $\sin^4 x - \cos^4 x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$. [5 marks]

Answer $x = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $\sin^4 x - \cos^4 x$ is a difference of two squares.
- Hint 2** Factorise it as $(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$. The second bracket is 1.
- Hint 3** $\sin^2 x - (1 - \sin^2 x) = \frac{1}{2}$, so $\sin^2 x = \frac{3}{4}$ and $\sin x = \pm \frac{\sqrt{3}}{2}$: $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can prove an identity by starting with one side and changing it into the other. ☐ ☐ ☒

I can use $\sin^2 x + \cos^2 x = 1$ to write an equation in one function, then solve it. ☐ ☐ ☒

I can turn a $\sin x = b \cos x$ into an equation in $\tan x$. ☐ ☐ ☒

I can find $\sin x$ and $\tan x$ exactly from $\cos x$ and the quadrant. ☐ ☐ ☒