

Trigonometry in two dimensions

About 85 minutes Calculator allowed 62 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-trigonometry-in-two-dimensions/

Date _____

A Do Now

The first is what today's lesson needs. The others bring back earlier topics.

A1 Needed today [2 marks] **A2** Last lesson [2 marks]

Write $\frac{12}{\sqrt{3}}$ in the form $a\sqrt{3}$,
where a is an integer. Do not
use a calculator.

Answer

PQ and PR are tangents to a
circle with centre O , touching it
at Q and R . Angle $QPR = 2x$.
Prove that angle
 $QOR = 180^\circ - 2x$.

A3 From chapter 5 [2 marks] **A4** From chapter 4 [2 marks]

Work out the distance between
the points $(1, -3)$ and $(5, 5)$. Give
your answer as a surd in its
simplest form.

Answer

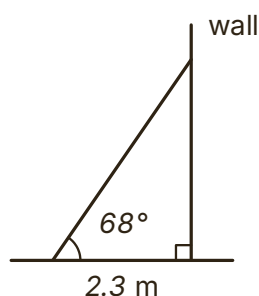
Work out the value of $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$.
Give your answer as a fraction.

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

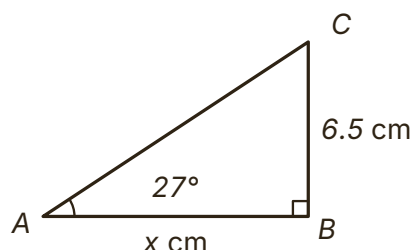
A ladder leans against a vertical wall. The foot of the ladder is 2.3 m from the wall. The ladder makes an angle of 68° with the horizontal ground. Work out the length of the ladder. Give your answer to 3 significant figures.



Not drawn accurately

Hint The 2.3 m is adjacent to the 68° and the ladder is the hypotenuse, so use cos. The unknown is on the bottom of the ratio.

B1 Work out the length x . Give your answer to 3 significant figures. [3 marks]

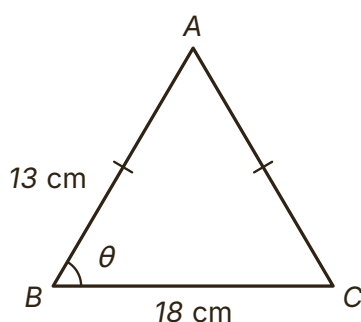


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Answer $x = \dots\dots$ cm

EXAMPLE 2

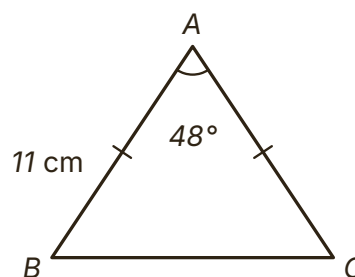
Triangle ABC is isosceles with $AB = AC = 13$ cm and $BC = 18$ cm. Work out the size of angle θ . Give your answer to 1 decimal place.



Not drawn accurately

Hint The line of symmetry from A meets BC at its midpoint, at right angles. That makes a right-angled triangle with hypotenuse 13 and base 9.

B2 Triangle ABC is isosceles with $AB = AC = 11$ cm and angle $BAC = 48^\circ$. Work out the length of BC . Give your answer to 3 significant figures. [3 marks]

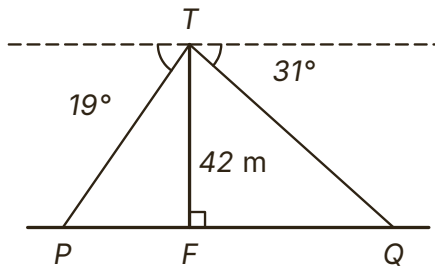


Not drawn accurately

Answer $BC = \dots\dots$ cm

EXAMPLE 3

FT is a vertical lighthouse 42 m tall, standing on level ground. Two boats P and Q are on opposite sides of the lighthouse. From T , the angle of depression of P is 19° and the angle of depression of Q is 31° . Work out the distance PQ . Give your answer to 3 significant figures.

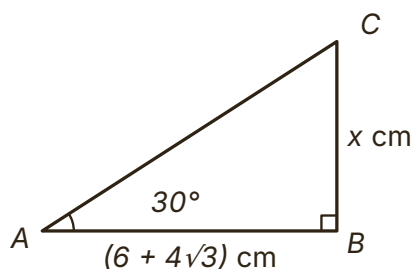


Not drawn accurately

Hint An angle of depression is measured down from the horizontal. It equals the angle of elevation at the boat (alternate angles). Find FP and FQ separately.

EXAMPLE 4

Work out the exact value of x . Give your answer in the form $p + q\sqrt{3}$, where p and q are integers. Do not use a calculator.

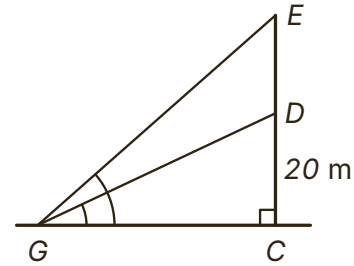


Not drawn accurately

Hint Half an equilateral triangle of side 2 has sides 1, $\sqrt{3}$ and 2, so $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

B3 [4 marks]

CD is a building 20 m tall on level ground. DE is a vertical flagpole on top of it. From a point G on the ground, the angle of elevation of D is 38° and the angle of elevation of E is 52° . Work out the height of the flagpole. Give your answer to 3 significant figures.

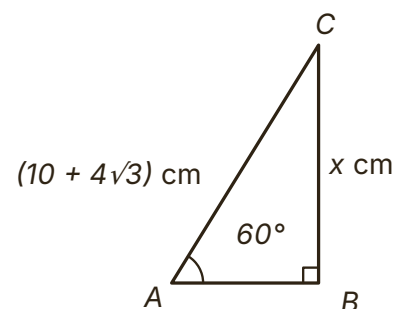


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Answer m

B4 [3 marks]

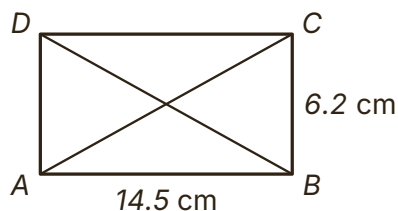
Work out the exact value of x . Give your answer in the form $p + q\sqrt{3}$, where p and q are integers. Do not use a calculator.



Not drawn accurately

Answer $x = \dots + \dots\sqrt{3}$

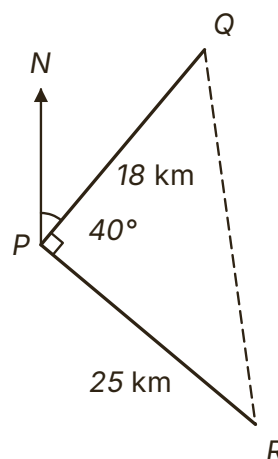
- C1** $ABCD$ is a rectangle with $AB = 14.5$ cm and $BC = 6.2$ cm. Work out the obtuse angle between the diagonals AC and BD . Give your answer to 1 decimal place.



Not drawn accurately

Answer°

- C2** Q is 18 km from P on a bearing of 040° . R is 25 km from P . Angle $QPR = 90^\circ$, with R to the south-east of P . Work out the distance QR , to 3 significant figures, and the bearing of Q from R , to the nearest degree.



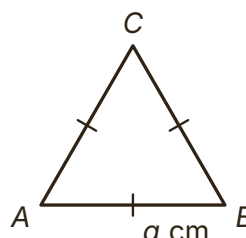
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Answer $QR = \dots\dots$ km bearing°

- C3** Work out the exact value of $\frac{\sin 60^\circ + \tan 30^\circ}{\cos 30^\circ}$. Do not use a calculator.

Answer

- C4** Prove that the area of an equilateral triangle with sides of length a cm is $\frac{\sqrt{3}}{4}a^2$ cm².

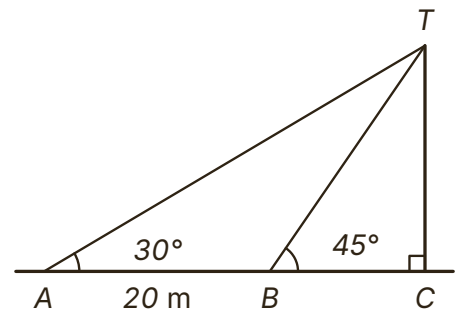


- D1** Ravi says, " $\sin 30^\circ + \sin 30^\circ = \sin 60^\circ$." [2 marks]

Show that Ravi is wrong. Do not use a calculator.

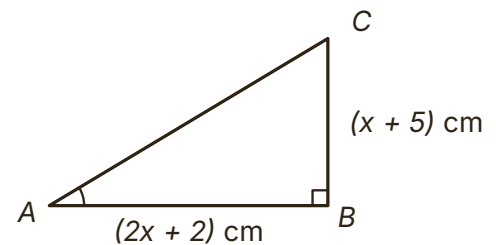
- D2** CT is a vertical mast on horizontal ground. A , B and C lie in a straight line, with $AB = 20$ m. The angle of elevation of T from A is 30° and from B is 45° . Work out the exact height CT . Give your answer in the form $p + q\sqrt{3}$, where p and q are integers. Do not use a calculator. You must show your working. [5 marks]

Answer $CT = \dots\dots + \dots\dots\sqrt{3}$ m



Not drawn accurately

- D3** ABC is a right-angled triangle. Angle $ABC = 90^\circ$ and $\tan(\text{angle } BAC) = \frac{3}{4}$. All lengths are in centimetres. Work out the perimeter of triangle ABC . [5 marks]



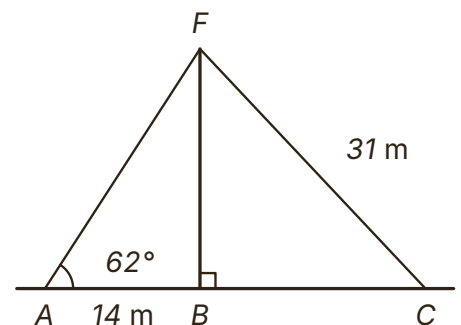
Answer cm Not drawn accurately

- D4** BF is a vertical post on horizontal ground. A , B and C are on the ground in a straight line. $AB = 14$ m and the angle of elevation of F from A is 62° . A straight cable FC is 31 m long.

- (a) Work out the height of the post BF . [2 marks]

Answer m

- (b) Hence work out the size of angle FCB . [2 marks]
Give your answer to 1 decimal place.



Not drawn accurately

Answer°

(c) Work out the distance AC .

[2 marks]

Answer m

Total for D4: 6 marks

D5 A and B are points on a circle with centre O and radius 7.5 cm. Angle $AOB = 112^\circ$.

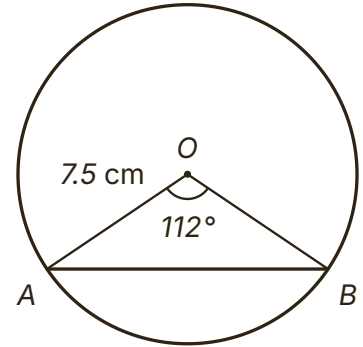
(a) Work out the length of the chord AB .

[3 marks]

Answer cm

(b) Work out the shortest distance from O to the chord AB .

[2 marks]



Answer cm

Total for D5: 5 marks

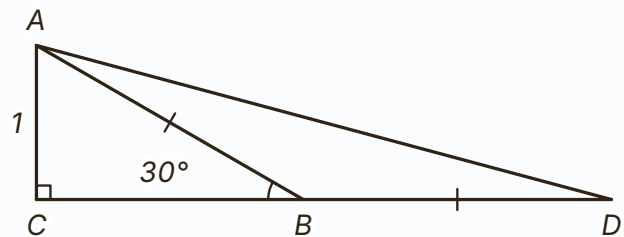
E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Triangle ABC has a right angle at C , $AC = 1$ and angle $ABC = 30^\circ$. The side CB is extended to D so that $BD = BA$. Use triangle ACD to show that $\tan 15^\circ = 2 - \sqrt{3}$. Do not use a calculator.

[5 marks]



Not drawn accurately

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find AB and CB using the exact values for 30° . Then look at triangle ABD : what kind of triangle is it?
- Hint 2** $AB = 2$ and $CB = \sqrt{3}$. Triangle ABD is isosceles, and angle $ABC = 30^\circ$ is an exterior angle of it, so angle $ADB = 15^\circ$.
- Hint 3** $CD = \sqrt{3} + 2$, so $\tan 15^\circ = \frac{1}{2 + \sqrt{3}}$. Multiply the top and bottom by $2 - \sqrt{3}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use sin, cos and tan to find a side or an angle, in context.



I can use angles of elevation and depression, and bearings.



I can use the exact values for 30° , 45° and 60° without a calculator.

