

Area under a curve and rates of change

About 35 minutes Calculator allowed 35 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/gcse-area-under-a-curve-and-rates-of-change/

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

A1 Needed today [2 marks] **A2** Last lesson [2 marks]

A trapezium has area 44 cm^2 . Its parallel sides are 10 cm and 12 cm. Find the distance between the parallel sides.

Answer

A cyclist speeds up steadily from rest to 17 m/s in 8 s, travels at 17 m/s for 17 s, then slows steadily to rest in 8 s. Find the total distance travelled.

Answer

A3 From chapter 7 [2 marks] **A4** From chapter 14 [2 marks]

The point $(2, 4)$ is reflected in the line $x = 1$, then translated by the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the coordinates of the final image.

Answer

Simplify $(3x^3y^4)^3 \div 3x^2y^4$.

Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use 3 equal strips to estimate the area under $y = 2x^2 + 1$ from $x = 0$ to $x = 3$. Is it an over- or underestimate?

Hint Find y at $x = 0, 1, 2, 3$; add three trapezia.

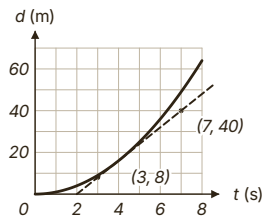
B1 Use 4 equal strips to estimate [4 marks]

the area under $y = x^2 + 4x$ from $x = 0$ to $x = 2$. Is it an over- or underestimate? Give a reason.

Answer

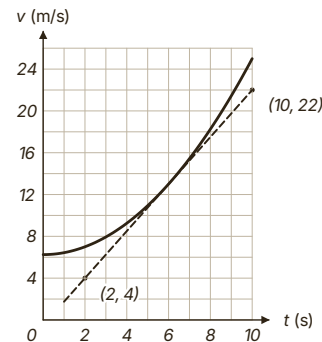
EXAMPLE 2

A tangent is drawn to a distance-time graph at $t = 4$ s. It passes through $(3, 8)$ and $(7, 40)$. Estimate the speed at $t = 4$ s.



Hint The tangent's gradient is the rate of change there.

- B2** A tangent is drawn to a velocity-time graph at $t = 6$ s. It passes through $(2, 4)$ and $(10, 22)$. Estimate the acceleration at $t = 6$ s. [2 marks]



Answer

c Practice

- C1** A curve has heights 4, 9, 16 and 25 at $x = 0, 1, 2, 3$. Use 3 strips to estimate the area under it. [2 marks]
- C2** A tank holds $V = 40 + 3t^2$ litres after t minutes. Find the average rate of flow from $t = 1$ to $t = 4$. [2 marks]
- C3** A curve bends downwards. Is a trapezium estimate of its area too big or too small? Why? [2 marks]
- C4** On a graph of depth h cm after t min, a tangent goes through $(0, 30)$ and $(12, 6)$. Find the rate h falls. [2 marks]

Answer

Answer

Answer

D Exam-style questions

- D1** The table gives the velocity of a train at times during the first 40 seconds after it leaves a station.

Time (s)	0	10	20	30	40
Velocity (m/s)	0	6	14	24	36

- (a) Use 4 strips of equal width to estimate the distance the train travels in the first 40 seconds. [3 marks]

Answer

- (b) The velocity rises by more in each 10 seconds than in the 10 before. Is your answer to (a) an overestimate or an underestimate? Give a reason. [1 mark]

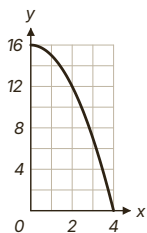
- (c) Use your answer to part (a) to estimate the average speed of the train over the 40 seconds. [2 marks]

Answer

Total for D1: 6 marks

- D2** Ali uses 4 strips of equal width to estimate the area under $y = 16 - x^2$ between $x = 0$ and $x = 4$. He gets 42. He says, "The true area is less than 42, because the curve slopes downwards." [4 marks]

Show that 4 strips give 42, and explain why Ali is wrong.



E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Two strips of equal width are used to estimate the area under $y = x^2 + k$ between $x = 0$ and $x = 4$. The estimate is 52. Find the value of k . [3 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write the three heights in terms of k .
- Hint 2** The heights are k , $4 + k$ and $16 + k$, and each strip is 2 wide.
- Hint 3** $2 \times \left(\frac{k}{2} + 4 + k + \frac{16 + k}{2} \right) = 4k + 24 = 52$, so $k = 7$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can estimate the area under a curve with trapezium strips and say if it is too big or too small. ☐ ☐ ☒
- I can find a rate of change from a chord or from a tangent. ☐ ☐ ☒