Hints online, answers free with an account:
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Grade 8/9 mixed revision 11

55 marks

Date

X Exam questions

Edexcel GCSE Higher style. Show all your working. The hints are at the end of the sheet. Use one at a time.

X1 Prove that $\frac{3+\sqrt{7}}{3-\sqrt{7}} + \frac{3-\sqrt{7}}{3+\sqrt{7}}$ is an integer. [4 marks]

You must show all your working.

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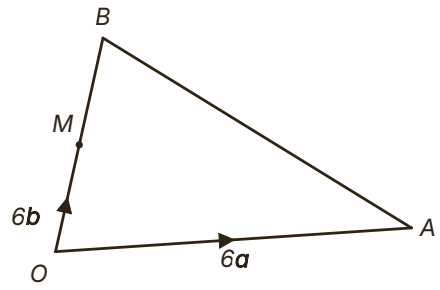
X2 OAB is a triangle. [4 marks]

$\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$.

M is the midpoint of OB.

The point P is such that $\overrightarrow{MP} = 4\mathbf{a} - \mathbf{b}$.

Prove that P lies on AB, and find the ratio $AP : PB$.



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X3 A, B and C are points on a circle. [4 marks]

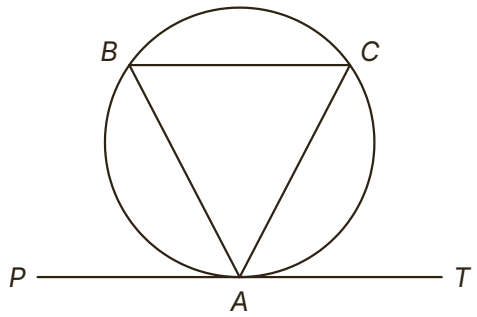
PAT is the tangent to the circle at A.

BC is parallel to PT.

Prove that triangle ABC is isosceles.

Give a reason for each stage of your working.

Diagram NOT accurately drawn



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X4 $x = 3$, $y = 6$ is one solution of the simultaneous equations

$$y = x^2 + px - 6$$

$$y = 2x$$

where p is a constant.

(a) Find the value of p. [1 mark]

Answer $p = \ldots$

(b) Find the other solution. [3 marks]

Answer $x = \ldots, y = \ldots$

Total for X4: 4 marks

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X5 Simplify fully $\dfrac{x^2 - 25}{3x + 6} \div \dfrac{x^2 - 2x - 15}{x^2 + 2x}$ [4 marks]

Answer .....

X6 $f(x) = x^2 + px + q$, where p and q are integers. [4 marks]
The graph of $y = f(x + 2)$ has a turning point at $(1, -5)$.
Find the value of p and the value of q.

Answer $p = \ldots$, $q = \ldots$

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X7 You must show all your working.

(a) Show that $(x+2)^3 - (x-2)^3 \equiv 12x^2 + 16$ [3 marks]

(b) Hence prove that, when x is an even number, $(x+2)^3 - (x-2)^3$ is a multiple of 16. [2 marks]

Total for X7: 5 marks

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X8 Find the set of values of x for which [5 marks]

$$2x - 3 < x^2 - 2x - 3 < 2x + 9$$

You must show all your working.

Answer .....

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X9 The function f is defined by [5 marks]

$$f(x) = x^2 - 4x + 1$$

Solve $f(2x) = f(x + 3)$.
You must show all your working.

Answer $x = \ldots$

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X10 A train journey has two stages. [5 marks]

Stage 1: 36 km, correct to the nearest km, in 40 minutes, correct to the nearest minute.

Stage 2: 25 km, correct to the nearest km, in 22 minutes, correct to the nearest minute.

Work out the lower bound for the average speed of the train for the whole journey.

Give your answer in km/h, correct to 1 decimal place.

Answer ... km/h

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X11 The equation $x^3 - 2x - 7 = 0$ has exactly one real solution.

(a) Find two consecutive integers between which the solution lies. [2 marks]
You must show how you got your answer.

Answer ... and ...

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{2x_n + 7}$ with x_0 equal to the smaller of your two integers to find x_3 [3 marks]
Give your answer correct to 4 decimal places.

Answer $x_3 = \ldots$

Total for X11: 5 marks

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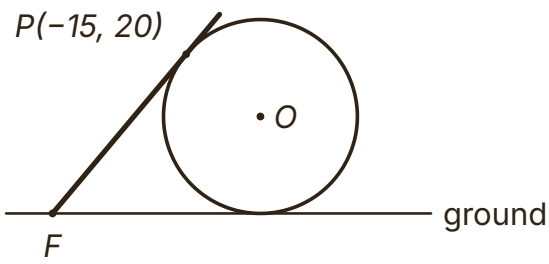
X12 The diagram shows the cross-section of a pipe resting on horizontal ground.
The cross-section is modelled by the circle with equation $x^2 + y^2 = 625$, where the units are centimetres.
The ground is the line $y = -25$.
A straight plank touches the pipe at the point $P(-15, 20)$ and rests on the ground at the point F.
Diagram NOT accurately drawn



(a) Find an equation of the line along the plank. [3 marks]

Answer ...

(b) Find the coordinates of F. [1 mark]

Answer $(\ldots, \ldots)$

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(c) Work out the length of the plank from F to P. [2 marks]

Answer ... cm

Total for X12: 6 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove results with vectors, algebra and circle theorems, giving a reason for each step.	○	○	○
I can find tangents to circles and solve non-linear simultaneous equations and quadratic inequalities.	○	○	○
I can work with composite and inverse functions, algebraic fractions, surds and transformations of graphs.	○	○	○
I can use bounds with division and subtraction, and find roots by iteration.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Write the two fractions over a common denominator, $(3-\sqrt{7})(3+\sqrt{7})$.

Hint 2 The common denominator is $9-7=2$. The numerator is $(3+\sqrt{7})^2+(3-\sqrt{7})^2$.

Hint 3 Expand both squares. The $\sqrt{7}$ terms cancel, leaving a whole number to divide by 2.

X2

Hint 1 M is the midpoint of OB, so $\overrightarrow{OM}=3\mathbf{b}$.

Hint 2 Find $\overrightarrow{OP}=\overrightarrow{OM}+\overrightarrow{MP}$, then $\overrightarrow{AP}=\overrightarrow{AO}+\overrightarrow{OP}$.

Hint 3 $\overrightarrow{AP}=-2\mathbf{a}+2\mathbf{b}$. Write it as a fraction of $\overrightarrow{AB}$; that fraction tells you how far along AB the point P is.

X3

Hint 1 Let angle $TAC=x$. Which angle in the triangle is equal to it by the alternate segment theorem?

Hint 2 BC is parallel to PT and AC crosses both lines. Which angle at C is equal to angle TAC?

Hint 3 Angle ABC and angle ACB are both equal to angle TAC. What does that tell you about the sides AB and AC?

X4

Hint 1 For (a), the values $x=3$ and $y=6$ make the first equation true. Substitute them.

Hint 2 For (b), use your value of p and set x^2+px-6 equal to $2x$.

Hint 3 $x^2-x-6=0$. You already know $x=3$ is a root, so $(x-3)$ is a factor. Find the other factor, then y.

X5

Hint 1 To divide by a fraction, multiply by its reciprocal.

Hint 2 Factorise all four expressions: x^2-25, $3x+6$, x^2+2x and $x^2-2x-15$.

Hint 3 $x^2-2x-15=(x-5)(x+3)$ and $x^2+2x=x(x+2)$. Cancel $(x-5)$ and $(x+2)$.

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X6	**Hint 1**	$f(x+2)$ is the graph of $y=f(x)$ moved 2 units to the left. Work backwards to find the turning point of $y=f(x)$.
	Hint 2	Move $(1,-5)$ two units to the right. A quadratic $x^2+\ldots$ with turning point (h,k) is $(x-h)^2+k$.
	Hint 3	$f(x)=(x-3)^2-5$. Expand and compare with x^2+px+q.
X7	**Hint 1**	Expand $(x+2)^2$ first, then multiply by $(x+2)$ again. Do the same for $(x-2)^3$.
	Hint 2	When you subtract $(x-2)^3$, the x^3 and $12x$ terms cancel.
	Hint 3	For part (b), write an even number as $x=2m$ and substitute into $12x^2+16$. Then take out a factor of 16.
X8	**Hint 1**	Split it into two inequalities: $2x-3<x^2-2x-3$ and $x^2-2x-3<2x+9$.
	Hint 2	The first becomes $x(x-4)>0$ and the second becomes $(x-6)(x+2)<0$.
	Hint 3	The first gives $x<0$ or $x>4$; the second gives $-2<x<6$. Draw both on one number line and find where they overlap.
X9	**Hint 1**	Replace every x in $f(x)$ with $2x$. Then do the same with $x+3$.
	Hint 2	$f(2x)=4x^2-8x+1$ and $f(x+3)=x^2+2x-2$.
	Hint 3	Set them equal and collect the terms to get $3x^2-10x+3=0$, then factorise.
X10	**Hint 1**	Average speed for the whole journey is the total distance divided by the total time. Do not average the two speeds.
	Hint 2	For the least average speed, use the least total distance and the greatest total time.
	Hint 3	The total distance is at least $35.5+24.5$ km and the total time is at most 63 minutes. Change the time to hours before dividing.

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X11

Hint 1 Substitute $x = 1, 2, 3, \ldots$ into $x^3 - 2x - 7$ and look for a change of sign.

Hint 2 $f(2)$ is negative and $f(3)$ is positive, so start the iteration with $x_0 = 2$.

Hint 3 $x_1 = \sqrt[3]{2 \times 2 + 7} = \sqrt[3]{11}$. Continue with the full values on your calculator until you reach x_3.

X12

Hint 1 The plank is a tangent to the circle at P, so it is perpendicular to the radius OP.

Hint 2 The gradient of OP is $-\frac{4}{3}$, so the plank has gradient $\frac{3}{4}$. Its line meets the ground where $y = -25$.

Hint 3 Substitute $y = -25$ into the plank's equation to find F. Then use Pythagoras with the horizontal and vertical distances between F and P.

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