Hints online, answers free with an account:
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Grade 8/9 mixed revision 4

63 marks

Date

X Exam questions

Edexcel GCSE Higher style. Show all your working. The hints are at the end of the sheet. Use one at a time.

X1 Show that $\frac{x+2}{x-1} - \frac{x-2}{x+1}$ can be written as $\frac{6x}{x^2-1}$ [3 marks]

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X2 The point $P(-2, 7)$ lies on the circle with equation $x^2 + y^2 = 53$. [3 marks]
Show that the line with equation $7y = 2x + 53$ is the tangent to the circle at P.

X3 The line with equation $y = x + 5$ meets the circle with equation $x^2 + y^2 = 37$ at the points A and B. [5 marks]
Find the length of AB.
Give your answer in the form $a\sqrt{2}$, where a is an integer.

Answer $AB = \ldots$

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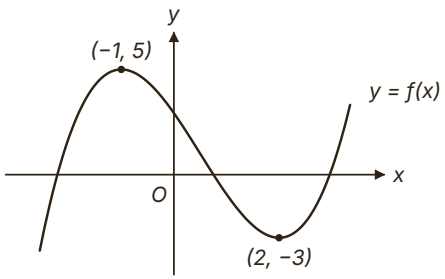
X4 The diagram shows a sketch of the graph of $y = f(x)$.
The graph has a maximum point at $(-1, 5)$ and a minimum point at $(2, -3)$.



(a) How many times does the graph of $y = f(x) - 4$ cross the x-axis? [2 marks]
Give a reason for your answer.

Answer ...

(b) Find the set of values of k for which the graph of $y = f(x) + k$ crosses the x-axis exactly three times. [3 marks]

Answer $\cdots < k < \ldots$

Total for X4: 5 marks

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X5 A, B and T are points on a circle, centre O. [5 marks]

AB is a diameter of the circle.

PT is the tangent to the circle at T.

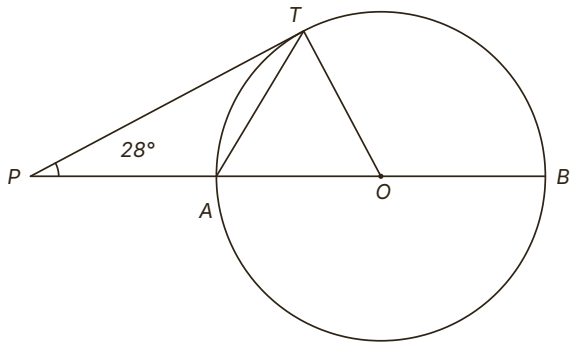
$PAOB$ is a straight line.

Angle $TPO = 28°$

Work out the size of angle ATP.

Give a reason for each stage of your working.

Diagram NOT accurately drawn



Answer Angle $ATP = \ldots°$

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X6 A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm. [6 marks]
The area of the rectangle is more than $30\ \text{cm}^2$.
The perimeter of the rectangle is less than 40 cm.
Find the range of possible values of x.
You must show all your working.

Answer .....

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X7 The function f is defined by

$$f(x) = \frac{6}{x+1} \qquad x \neq -1$$

(a) Show that $ff(x) = \dfrac{6(x+1)}{x+7}$ [3 marks]

(b) Hence solve $ff(x) = x$. [3 marks]

Answer $x = \ldots$

Total for X7: 6 marks

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X8 A cube is made of one metal. Each edge of the cube is 4.2 cm, correct to 1 decimal place. [6 marks]

The mass of the cube is 640 g, correct to the nearest 5 g.

The table shows the densities of four metals.

Jo says, "The cube is made of brass."

Use bounds to decide whether Jo can be sure. You must show all your working.

Metal	Density (g/cm^3)
Iron	7.87
Brass	8.50
Copper	8.96
Silver	10.49

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X9 $f(x) = x^3 + 3x - 20$

(a) Show that the equation $f(x) = 0$ has a solution between $x = 2$ and $x = 3$. [2 marks]

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{20 - 3x_n}$ with $x_0 = 2$ to find the value of x_4
Give your answer correct to 4 decimal places. [2 marks]

Answer $x_4 = \ldots$

(c) By choosing a suitable interval, show that the solution of $f(x) = 0$ is 2.35 correct to 2 decimal places. [2 marks]

Total for X9: 6 marks

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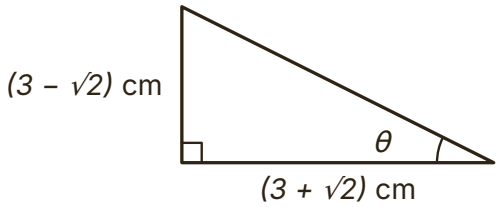
X10 The diagram shows a right-angled triangle.
The two shorter sides are $(3 + \sqrt{2})$ cm and $(3 - \sqrt{2})$ cm.
Diagram NOT accurately drawn



(a) Show that the length of the hypotenuse is $\sqrt{22}$ cm. [3 marks]

(b) Find $\tan\theta$. Give your answer in the form $\dfrac{a + b\sqrt{2}}{c}$, where a, b and c are integers. [3 marks]

Answer $\tan\theta = \ldots$

Total for X10: 6 marks

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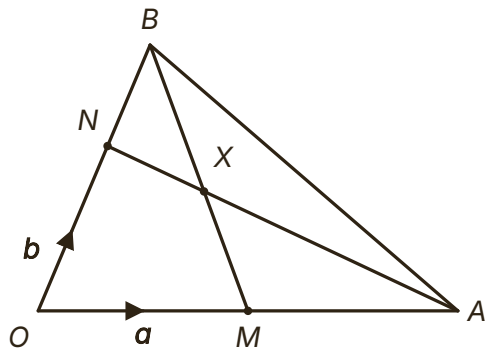
X11 OAB is a triangle.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is the midpoint of OA.
N is the point on OB such that $ON : NB = 2 : 1$.
The lines AN and BM meet at the point X.
Diagram NOT accurately drawn



(a) Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [5 marks]
You must show all your working.

Answer $\overrightarrow{OX} = \ldots$

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(b) Find the ratio $AX : XN$. [1 mark]

Answer ⋯ : ⋯

Total for X11: 6 marks

X12 Here are the first five terms of a sequence.

$$4 \qquad 10 \qquad 18 \qquad 28 \qquad 40$$

(a) Find an expression, in terms of n, for the nth term of the sequence. [2 marks]

Answer ⋯

(b) Prove that every term of the sequence is even. [2 marks]

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(c) Prove that the sum of any two consecutive terms of the sequence is 4 less than twice a square number. [2 marks]

Total for X12: 6 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove results with vectors, algebra and circle theorems, giving a reason for each step.	○	○	○
I can find tangents to circles and solve non-linear simultaneous equations and quadratic inequalities.	○	○	○
I can work with composite and inverse functions, algebraic fractions, surds and transformations of graphs.	○	○	○
I can use bounds with division and subtraction, and find roots by iteration.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Use the common denominator $(x-1)(x+1)$.
	Hint 2	The numerator is $(x+2)(x+1)-(x-2)(x-1)$. Expand each product.
	Hint 3	$(x-2)(x-1)=x^2-3x+2$. The minus sign in front changes the sign of every term: $-x^2+3x-2$.
X2	**Hint 1**	Two things need showing: the line passes through P, and it is perpendicular to the radius OP.
	Hint 2	Rearrange $7y=2x+53$ to find its gradient. Find the gradient of OP from $O(0,0)$ to $P(-2,7)$.
	Hint 3	The gradients are $\frac{2}{7}$ and $-\frac{7}{2}$. Multiply them and say what the result tells you.
X3	**Hint 1**	Find the coordinates of A and B first by substituting $y=x+5$ into the circle's equation.
	Hint 2	You should get $x^2+5x-6=0$. Solve it and use $y=x+5$ to find both points.
	Hint 3	The points are $(1,6)$ and $(-6,-1)$. Use Pythagoras with the differences in their coordinates, then simplify the surd.
X4	**Hint 1**	$y=f(x)+k$ is the same curve moved up by k (down if k is negative). Each turning point moves up by k.
	Hint 2	For (a), move both turning points down 4 and look at which side of the x-axis each one is on.
	Hint 3	For three crossings the maximum must be above the axis and the minimum below it: $5+k>0$ and $-3+k<0$. Solve both inequalities and put them together.

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X5

Hint 1 OT is a radius and PT is a tangent. What is angle OTP?

Hint 2 Find angle TOP from triangle OTP. Then look at triangle OAT : which two sides are equal?

Hint 3 Triangle OAT is isosceles with $OA = OT$, so the angles at A and T are each $(180 - \text{angle } TOA) \div 2$. Angle ATP is what is left of the $90°$ at T.

X6

Hint 1 Write one inequality for the area and one for the perimeter.

Hint 2 The area gives $x^2 + 3x - 40 > 0$. Factorise it. The perimeter is $2(x + 5) + 2(x - 2)$.

Hint 3 The area condition gives $x < -8$ or $x > 5$, but the width $x - 2$ must be positive. The perimeter gives $4x + 6 < 40$.

X7

Hint 1 $ff(x)$ means put $f(x)$ into f: replace x in $\dfrac{6}{x+1}$ with $\dfrac{6}{x+1}$.

Hint 2 Write the denominator $\dfrac{6}{x+1} + 1$ as one fraction, $\dfrac{x+7}{x+1}$, then divide.

Hint 3 For (b), multiply both sides of $\dfrac{6(x+1)}{x+7} = x$ by $x + 7$ and collect the terms to get $x^2 + x - 6 = 0$.

X8

Hint 1 Write down the bounds for the edge and for the mass. The volume of a cube is the edge cubed.

Hint 2 Density is mass divided by volume. The smallest density uses the least mass and the greatest volume.

Hint 3 Work out $637.5 \div 4.25^3$ and $642.5 \div 4.15^3$. Then look at which metals have a density between these two values.

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X9

Hint 1 For part (a), work out $f(2)$ and $f(3)$ and look at their signs.

Hint 2 For part (b), start with $x_1 = \sqrt[3]{20 - 3 \times 2}$ and keep going. For part (c), find the two numbers that round to 2.35 at the ends of its rounding interval.

Hint 3 Any number from 2.345 up to 2.355 rounds to 2.35. Work out $f(2.345)$ and $f(2.355)$ and show that the sign changes between them.

X10

Hint 1 Use Pythagoras: square each of the two shorter sides. $(3 + \sqrt{2})^2$ means $(3 + \sqrt{2})(3 + \sqrt{2})$.

Hint 2 When you add the two squares, the $\sqrt{2}$ terms cancel. For (b), the side opposite θ is $(3 - \sqrt{2})$ cm.

Hint 3 $\tan\theta = \dfrac{3 - \sqrt{2}}{3 + \sqrt{2}}$. Multiply the top and the bottom by $3 - \sqrt{2}$; the denominator is $9 - 2 = 7$.

X11

Hint 1 Write $\overrightarrow{AN}$ and $\overrightarrow{BM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ first. $\overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ and $\overrightarrow{OM} = \frac{1}{2}\mathbf{a}$.

Hint 2 Use two different numbers: $\overrightarrow{OX} = \overrightarrow{OA} + \lambda\,\overrightarrow{AN}$ and $\overrightarrow{OX} = \overrightarrow{OB} + \mu\,\overrightarrow{BM}$. Both are the same vector.

Hint 3 Make the coefficients of $\mathbf{a}$ equal and the coefficients of $\mathbf{b}$ equal: $1 - \lambda = \frac{1}{2}\mu$ and $\frac{2}{3}\lambda = 1 - \mu$. Solve these simultaneous equations and put λ back in.

X12

Hint 1 Find the differences between the terms, then the second differences.

Hint 2 A second difference of 2 means the nth term starts n^2. For part (b), factorise your nth term.

Hint 3 For part (c), add the nth term and the $(n+1)$th term to get $2n^2 + 8n + 4$, then take out 2 and complete the square inside the bracket.

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