Hints online, answers free with an account:
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Grade 8/9 mixed revision 6

52 marks

Date

X Exam questions

Edexcel GCSE Higher style. Show all your working. The hints are at the end of the sheet. Use one at a time.

X1 The functions f and g are such that [3 marks]

$$f(x) = 3x - 5 \qquad g(x) = x^2 + 2$$

Solve $fg(x) = 22$.
Give your answers in surd form.

Answer $x = \ldots$

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X2 A, B, C and D are points on a circle. [3 marks]

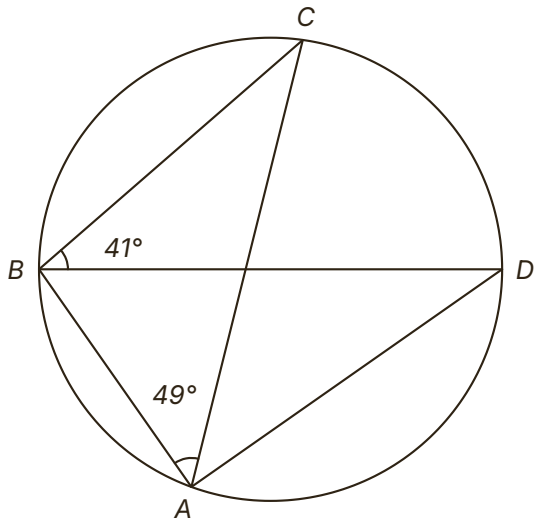
Angle $DBC = 41°$

Angle $BAC = 49°$

Prove that BD is a diameter of the circle.

Give a reason for each stage of your working.

Diagram NOT accurately drawn



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X3 The curve with equation $xy = 20$ and the line with equation $y = 3x - 7$ meet at two points. [3 marks]
Find the coordinates of the two points.

Answer $(\ldots, \ldots)$ and $(\ldots, \ldots)$

X4 Solve $x^2 - 6x + 4 < 0$ [3 marks]
Give your answer in the form $a - \sqrt{b} < x < a + \sqrt{b}$, where a and b are integers.

Answer .....

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X5 [4 marks]

$p = 8.4$ correct to 1 decimal place.
$q = 5.7$ correct to 1 decimal place.
$r = 2.36$ correct to 2 decimal places.

$$y = \frac{p}{q - r}$$

Work out the upper bound for y. Give your answer correct to 4 significant figures.
You must show all your working.

Answer $y = \ldots$

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X6 The iteration formula $x_{n+1} = \sqrt{7 - \dfrac{3}{x_n}}$ is used to find an estimate of a solution of an equation. [4 marks]

The equation can be written in the form $x^3 + ax + b = 0$, where a and b are integers.

Find the value of a and the value of b.

Answer $a = \ldots$ $b = \ldots$

X7 Prove that $n^3 - n$ is a multiple of 6 for every integer n. [4 marks]

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X8 You must show all your working.

(a) Show that $(\sqrt{6}+\sqrt{2})^2 = 8+4\sqrt{3}$. [2 marks]

(b) Hence write $\dfrac{8}{(\sqrt{6}+\sqrt{2})^2}$ in the form $a+b\sqrt{3}$, where a and b are integers. [3 marks]

Answer ...

Total for X8: 5 marks

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X9 Two tangents to the circle with equation $x^2 + y^2 = 80$ are parallel to the line with equation $y = 2x + 7$. [5 marks]

Find the equations of these two tangents.

You must show all your working.

Answer $y = \ldots$ and $y = \ldots$

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X10 Prove that [6 marks]

$$\frac{x^4 - 81}{x^2 - 9} - \frac{6x^2 - 18x}{x - 3}$$

is positive for every value of x for which it is defined.

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X11 $f(x) = x^2 - 2x - 8$

The graph of $y = f(x)$ is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and then reflected in the x-axis.

The new graph has equation $y = g(x)$.

(a) Find the coordinates of the points where the graph of $y = g(x)$ crosses the x-axis. [2 marks]

Answer $(\ldots, 0)$ and $(\ldots, 0)$

(b) Find the coordinates of the turning point of $y = g(x)$, and state whether it is a maximum or a minimum. [2 marks]

Answer $(\ldots, \ldots)$, ...

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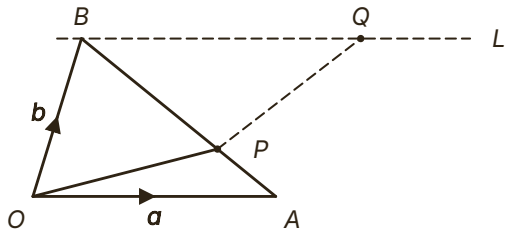
(c) Show that $g(x) = -x^2 + 8x - 7$. [2 marks]

Total for X11: 6 marks

X12 OAB is a triangle.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
P is the point on AB such that $AP : PB = 1 : 3$.
L is the straight line through B that is parallel to OA.
The line OP is extended to meet L at the point Q.
Diagram NOT accurately drawn



(a) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Give your answer in its simplest form. [2 marks]

Answer $\overrightarrow{OP} = \ldots$

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(b) Find $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3 marks]

Answer $\overrightarrow{OQ} = \ldots$

(c) Find the ratio $OP : PQ$. [1 mark]

Answer $\cdots : \ldots$

Total for X12: 6 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove results with vectors, algebra and circle theorems, giving a reason for each step.	○	○	○
I can find tangents to circles and solve non-linear simultaneous equations and quadratic inequalities.	○	○	○
I can work with composite and inverse functions, algebraic fractions, surds and transformations of graphs.	○	○	○
I can use bounds with division and subtraction, and find roots by iteration.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

- **Hint 1** $fg(x)$ means do g first, then f.
- **Hint 2** $fg(x) = 3(x^2 + 2) - 5$. Simplify it.
- **Hint 3** Solve $3x^2 + 1 = 22$ to find x^2. Taking the square root gives two answers.

X2

- **Hint 1** Angles DBC and DAC both stand on the arc DC.
- **Hint 2** Find angle DAC, then add it to angle BAC to get angle BAD.
- **Hint 3** If a chord makes an angle of $90°$ at the circumference, the chord must be a diameter. Show angle BAD is $90°$ and say which theorem you are using.

X3

- **Hint 1** Replace y in $xy = 20$ by $3x - 7$.
- **Hint 2** Expand $x(3x - 7) = 20$ and rearrange to $3x^2 - 7x - 20 = 0$.
- **Hint 3** Factorise as $(3x + 5)(x - 4) = 0$. Find y for each value of x, taking care with the negative fraction.

X4

- **Hint 1** The quadratic does not factorise. Find the critical values by completing the square or by the formula.
- **Hint 2** $x^2 - 6x + 4 = (x - 3)^2 - 5$.
- **Hint 3** $(x - 3)^2 = 5$ gives $x = 3 \pm \sqrt{5}$. The quadratic is less than 0 between these two values.

X5

- **Hint 1** Write down the upper and lower bounds of p, q and r.
- **Hint 2** To make a fraction as large as possible, make the top as large as possible and the bottom as small as possible.
- **Hint 3** The smallest value of $q - r$ uses the smallest q and the largest r: $5.65 - 2.365$.

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X6

Hint 1 When the iteration settles, x_{n+1} and x_n are the same number. Replace both with x.

Hint 2 Square both sides to remove the square root.

Hint 3 $x^2 = 7 - \dfrac{3}{x}$. Multiply every term by x, then move all the terms to one side so that it equals 0; watch the signs.

X7

Hint 1 Take out the common factor n.

Hint 2 $n^2 - 1$ is a difference of two squares. Write $n^3 - n$ as a product of three brackets.

Hint 3 $(n-1)n(n+1)$ is a product of three consecutive integers. What can you say about how many of them are even, and how many are multiples of 3?

X8

Hint 1 Expand $(\sqrt{6}+\sqrt{2})(\sqrt{6}+\sqrt{2})$. The middle terms are both $\sqrt{12}$.

Hint 2 For (b), replace $(\sqrt{6}+\sqrt{2})^2$ with $8+4\sqrt{3}$. You can divide the top and the bottom by 4.

Hint 3 $\dfrac{2}{2+\sqrt{3}}$: multiply the top and the bottom by $2-\sqrt{3}$.

X9

Hint 1 Both tangents have gradient 2. Each one touches the circle where the radius is perpendicular to it.

Hint 2 The radius to each point of contact lies on the line $y = -\dfrac{1}{2}x$. Find where this line meets the circle; there are two points.

Hint 3 The points of contact are $(8, -4)$ and $(-8, 4)$. Write the line with gradient 2 through each point.

X10

Hint 1 $x^4 - 81$ is a difference of two squares: $(x^2)^2 - 9^2$.

Hint 2 Cancel to get $x^2 + 9$ from the first fraction. Factorise $6x^2 - 18x$ as $6x(x-3)$ and cancel.

Hint 3 The expression becomes $x^2 - 6x + 9$. Write it as a square, then ask when that square can be 0.

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X11

Hint 1 Find where $y = f(x)$ crosses the x-axis and its turning point first. Then apply the translation, then the reflection.

Hint 2 A translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ adds 3 to every x-coordinate. A reflection in the x-axis changes the sign of every y-coordinate, so points on the x-axis stay where they are.

Hint 3 $f(x) = (x-1)^2 - 9$. For (c), the translation gives $f(x-3)$ and the reflection puts a minus sign in front: $g(x) = -f(x-3)$. Expand carefully.

X12

Hint 1 P is a quarter of the way from A to B, so start at A and add $\frac{1}{4}\overrightarrow{AB}$.

Hint 2 Every point on line L is $\mathbf{b}$ plus some number of $\mathbf{a}$, because L goes through B and is parallel to OA. Every point on line OP is a multiple of $\overrightarrow{OP}$.

Hint 3 Set $k\left(\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}\right) = \mathbf{b} + t\mathbf{a}$. The $\mathbf{b}$ terms give $\frac{1}{4}k = 1$. Then $\overrightarrow{OQ}$ is k times $\overrightarrow{OP}$.

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