



Iteration

About 50 minutes Calculator allowed 44 marks

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

A1 Needed today [2 marks] **A2** From chapter 18 [2 marks]

The equation $x^3 + 2x = 54$ has one solution. Show that it lies between two consecutive whole numbers, and find them.

Answer

In a histogram, the bar for $0 < x \leq 10$ is 5 cm tall and stands for 24 people. The bar for $10 < x \leq 35$ is 8 cm tall. How many people does it stand for?

Answer

A3 From chapter 23 [2 marks] **A4** From chapter 24 [2 marks]

The curve $y = ab^x$ passes through $(2, 16)$ and $(3, 32)$, where $b > 0$. Find a and b .

Answer

Make x the subject of $9x - a = bx + 11$.

Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use $x_{n+1} = \sqrt{3x_n + 11}$ with $x_1 = 4$ to find x_2 , x_3 and x_4 , correct to 4 decimal places.

Hint Put each answer back into the formula. Keep the full calculator value: only round what you write down.

B1 Use $x_{n+1} = \sqrt[3]{8x_n - 3}$ with $x_1 = 3$ to find x_2 , x_3 and x_4 , correct to 4 decimal places. [3 marks]

Answer $x_2 = \dots\dots$ $x_3 = \dots\dots$ $x_4 = \dots\dots$

EXAMPLE 2

Show that $x^3 + 3x - 20 = 0$ has a solution of 2.35, correct to 2 decimal places.

Hint Every number from 2.345 up to (not including) 2.355 rounds to 2.35. A sign change between those two ends puts the solution inside.

B2 Show that $x^3 - 5x - 11 = 0$ [3 marks]
has a solution of 2.95, correct to 2 decimal places.

c Practice

C1 Show that $x^3 - 6x - 3 = 0$ has a [2 marks]
solution between 2 and 3.

C2 Show that $x^3 - 3x^2 - 4 = 0$ can [2 marks]
be rearranged to give
 $x = 3 + \frac{4}{x^2}$.

C3 Use $x_{n+1} = \sqrt[3]{20 - 2x_n}$ with [3 marks]
 $x_1 = 2$. Keep going until two values in a row are the same when rounded to 3 decimal places. Write down that solution, correct to 3 decimal places.

C4 The values of $x_{n+1} = \sqrt{\frac{2x_n + 7}{3}}$ [2 marks]
settle on a solution of a quadratic equation. Write that equation in the form $ax^2 + bx + c = 0$, where a , b and c are whole numbers.

Answer

Answer

C5 Here are three rearrangements [2 marks]
of $x^3 - 5x + 1 = 0$. Which two are correct?

A: $x = \sqrt[3]{5x - 1}$ B: $x = \frac{x^3 - 1}{5}$

C: $x = \sqrt{5 - \frac{1}{x}}$

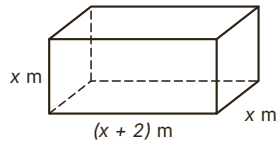
Answer

D Exam-style questions

D1 Sam solves $x^2 - 5x - 6 = 0$ with $x_{n+1} = \sqrt{5x_n + 6}$ and $x_1 = 1$. The values settle on 6. Sam says, "So 6 is the only solution." [2 marks]

Explain why Sam is wrong.

- D2** A water tank is a cuboid. It is x m wide, x m high and $(x + 2)$ m long. It holds 50 m^3 of water when it is full.



- (a) Show that $x^3 + 2x^2 - 50 = 0$. [1 mark]

- (b) Show that this equation can be rearranged to give $x = \sqrt{\frac{50}{x+2}}$. [2 marks]

- (c) Use $x_{n+1} = \sqrt{\frac{50}{x_n+2}}$ with $x_1 = 3$ to find x_4 , correct to 4 decimal places. [3 marks]

Answer $x_4 = \dots\dots\dots$

- (d) Keep going until two values in a row agree to 3 decimal places. Write down the width of the tank, correct to the nearest centimetre. [2 marks]

Answer $\dots\dots\dots$ cm

Total for D2: 8 marks

- D3** An iterative formula is $x_{n+1} = \frac{x_n^3 + 4}{10}$.

- (a) Use $x_1 = 0$ to find x_2 and x_3 . Give x_3 correct to 4 decimal places. [2 marks]

Answer $x_2 = \dots\dots\dots$ $x_3 = \dots\dots\dots$

- (b) Write down the equation that this formula is solving, in the form $x^3 + ax + b = 0$. [2 marks]

Answer $\dots\dots\dots$

- (c) Asha starts instead with $x_1 = 4$. Work out x_2 and explain what happens to the values after that. [1 mark]

Total for D3: 5 marks

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** An iterative formula is $x_{n+1} = \sqrt{ax_n + b}$, where a and b are numbers. Starting with $x_1 = 1$ gives $x_2 = 4$. The values settle on 6. Work out a and b . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Put $x_1 = 1$ and $x_2 = 4$ into the formula and square both sides.
- Hint 2** If the values settle on 6, then $6 = \sqrt{6a + b}$.
- Hint 3** You now have $a + b = 16$ and $6a + b = 36$. Solve them together.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can rearrange an equation into an iterative formula and run it. ☐ ☐ ☒
- I can use a sign change to show where a solution lies. ☐ ☐ ☒
- I can prove a solution is correct to 2 decimal places. ☐ ☐ ☒