



Properties of vectors

About 50 minutes No calculator 44 marks

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** From chapter 16 [2 marks]
- The point $(-6, 1)$ is reflected in the line $x = 3$. Find the coordinates of its image.
- Work out $\left(\frac{125}{64}\right)^{-\frac{1}{3}}$. Give your answer as a fraction or a whole number.

Answer

Answer

- A3** From chapter 22 [2 marks] **A4** From chapter 1 [2 marks]
- In triangle ABC , $BA = 7$ cm, $BC = 8$ cm and angle $ABC = 120^\circ$. Find the length AC . ($\cos 120^\circ = -\frac{1}{2}$)
- Find the smallest positive whole number k such that $180k$ is a square number.

Answer

Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$. Write $3\mathbf{a} - 2\mathbf{b}$ as a column vector.

Hint Multiply each vector first, then subtract the tops and subtract the bottoms.

- B1** $\mathbf{p} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$. [2 marks]
- Write $4\mathbf{p} + 3\mathbf{q}$ as a column vector.

Answer

EXAMPLE 2

$\mathbf{a} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Work out the length of $\mathbf{a} + 3\mathbf{b}$.

Hint Find the column vector first. Its length is the hypotenuse of a right-angled triangle with the top and bottom numbers as the other two sides.

B2 $\mathbf{u} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Work out the length of $\mathbf{u} + 2\mathbf{w}$. [3 marks]

Answer

c Practice

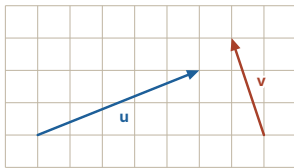
C1 A point A is translated by $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$. [2 marks]
Its image is $B(1, 3)$. Write down the coordinates of A .

Answer

C2 Simplify $3(2\mathbf{a} - \mathbf{b}) - 2(\mathbf{a} - 4\mathbf{b})$. [2 marks]

Answer

C3 The grid shows the vectors $\mathbf{u}$ and $\mathbf{v}$. Write $\mathbf{u} - 2\mathbf{v}$ as a column vector. [2 marks]



Answer

C4 $\begin{pmatrix} 6 \\ m \end{pmatrix}$ is parallel to $\begin{pmatrix} -4 \\ 10 \end{pmatrix}$. Work out the value of m . [2 marks]

Answer

C5 $\mathbf{a}$ and $\mathbf{b}$ are not parallel. [2 marks]
 $(p + 2)\mathbf{a} + 3\mathbf{b} = 7\mathbf{a} + (q - 1)\mathbf{b}$.
Work out p and q .

Answer $p = \dots\dots$ $q = \dots\dots$

C6 $\mathbf{u} + 2\mathbf{v} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$ and [3 marks]
 $\mathbf{u} - \mathbf{v} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$. Write $\mathbf{u}$ and $\mathbf{v}$ as column vectors.

Answer $\mathbf{u} = \dots\dots$ $\mathbf{v} = \dots\dots$

D Exam-style questions

- D1** Tom says, " $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 8 \end{pmatrix}$ are not parallel, because one is not a positive multiple of the other." [2 marks]
- Explain why Tom is wrong.

- D2** An aircraft flies with velocity $\begin{pmatrix} 120 \\ 50 \end{pmatrix}$ km/h in still air. The wind adds $\begin{pmatrix} -24 \\ 22 \end{pmatrix}$ km/h. The first number is east and the second is north.

- (a) Write the aircraft's resultant velocity as a column vector. [1 mark]

Answer

- (b) Work out the aircraft's speed. [2 marks]

Answer km/h

- (c) The aircraft starts at the point (10, 20), in km. Work out where it is after 30 minutes. [2 marks]

Answer

Total for D2: 5 marks

- D3** A is the point (1, 3), B is (5, 1) and C is (15, -4).

- (a) Write $\overrightarrow{AB}$ and $\overrightarrow{BC}$ as column vectors. [2 marks]

Answer $\overrightarrow{AB} = \dots\dots$ $\overrightarrow{BC} = \dots\dots$

- (b) Use your answers to explain why A , B and C lie on a straight line. [2 marks]

- (c) Work out the ratio $AB : BC$. [1 mark]

Answer

- (d) Work out the length of AC . Give your answer as a surd in its simplest form. [2 marks]

Answer

Total for D3: 7 marks

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $\mathbf{p} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$. The vector $\mathbf{p} + k\mathbf{q}$ has length 10. Work out the two possible values of k . [4 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write $\mathbf{p} + k\mathbf{q}$ as one column vector with k in it.
Hint 2 $\begin{pmatrix} 2k - 6 \\ -2k - 4 \end{pmatrix}$ has length 10, so
 $(2k - 6)^2 + (-2k - 4)^2 = 100$.
Hint 3 This gives $8k^2 - 8k - 48 = 0$, so $k^2 - k - 6 = 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can add, subtract and multiply column vectors. ☐ ☐ ☒
- I can work out the length of a vector. ☐ ☐ ☒
- I can decide whether two vectors are parallel. ☐ ☐ ☒