

Transformations of  $y = f(x)$ 

About 50 minutes No calculator 36 marks

Hints online, answers free with an account:  
mathswariz.uk/worksheets/gcse-transformations-of-y-f-x/

Date \_\_\_\_\_

**A Do Now**

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** From chapter 17 [2 marks]
- The point  $(3, -1)$  is rotated  $90^\circ$  clockwise about the origin, then reflected in the  $x$ -axis. Find the coordinates of the final image.
- List the integers  $x$  that satisfy  $x^2 - 15x + 50 < 0$ .

Answer .....

Answer .....

- A3** From chapter 23 [2 marks] **A4** From chapter 15 [2 marks]
- Find the equation of the tangent to  $x^2 + y^2 = 169$  at the point  $(5, -12)$ . Give it in the form  $y = mx + c$ .
- Solve  $6(x + 4) - 7(x - 2) = 36$ .

Answer .....

Answer .....

**B Example, then your turn**

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

The point  $(2, 5)$  lies on the graph of  $y = f(x)$ . Write down its image on the graph of  $y = f(x - 4)$ , on  $y = -f(x)$  and on  $y = f(x) + 3$ .

**Hint** Inside the bracket moves  $x$ , the opposite way to its sign. Outside moves  $y$ .

- B1** The point  $(-3, 4)$  lies on the graph of  $y = f(x)$ . Write down its image on the graph of  $y = f(x + 5)$ , on  $y = f(-x)$  and on  $y = f(x) - 6$ . [3 marks]

Answer  $(\dots, \dots)$   $(\dots, \dots)$   $(\dots, \dots)$

**EXAMPLE 2**

The graph of  $y = x^2$  is translated by  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ .  
Write down the equation of the new graph.

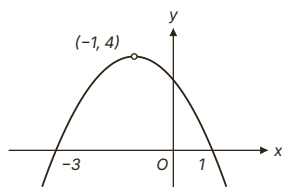
**Hint** Moving left 3 puts +3 inside the bracket.  
Moving up 2 adds 2 outside.

**B2** The graph of  $y = x^3$  is [2 marks]  
translated by  $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ . Write  
down the equation of the new  
graph.

Answer .....

**c Practice**

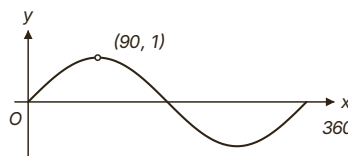
**C1** The sketch shows  $y = f(x)$ . [1 mark]  
Write down the turning point of  
 $y = f(x - 5)$ .



Not drawn accurately

Answer (....., .....)

**C2** The sketch shows  $y = \sin x$ , with [2 marks]  
its maximum point  $(90, 1)$ . Write  
down the maximum point of  
 $y = \sin x + 3$  and of  
 $y = \sin(x - 40)$ .



Answer (... , ...) and (... , ...)

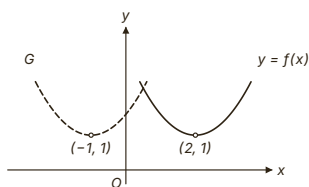
**C3** The graph of  $y = x^2 + 5x$  is [1 mark]  
reflected in the  $y$ -axis. Write  
down the equation of the new  
graph.

Answer .....

**C4** The graph of  $y = x^3$  is translated [2 marks]  
by  $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$ . Where does the new  
graph cross the  $x$ -axis?

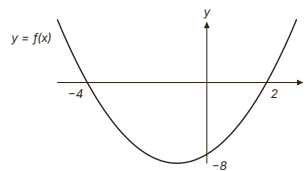
Answer .....

**C5** The sketch shows  $y = f(x)$  and a [1 mark]  
curve  $G$  of the same shape. Write  
down the equation of  $G$  in terms  
of  $f$ .



Answer .....

**C6** The sketch shows  $y = f(x)$ . [2 marks]  
Write down the points where  
the graph of  $y = f(-x)$  crosses  
the axes.



Not drawn accurately

Answer (... , ...) , (... , ...) , (... , ...)

## D Exam-style questions

- D1** Mia says, "The graph of  $y = f(x) - 3$  is the graph of  $y = f(x)$  moved 3 units to the left." [2 marks]

Explain why Mia is wrong, and describe the transformation fully.

- D2** A curve has equation  $y = x^2 - 14x + 40$ .

- (a) Write  $x^2 - 14x + 40$  in the form  $(x - a)^2 - b$ , where  $a$  and  $b$  are positive integers. [2 marks]

Answer .....

- (b) Hence describe fully the single transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 - 14x + 40$ . [2 marks]

Answer .....

- (c) The graph of  $y = x^2 - 14x + 40$  is reflected in the  $x$ -axis. Write down its equation and its turning point. [2 marks]

Answer  $y = \dots\dots\dots$  ( $\dots\dots, \dots\dots$ )

Total for D2: 6 marks

- D3** The point  $A(4, 9)$  lies on the graph of  $y = f(x)$ . Under the transformation to  $y = f(x + p) + q$ , the image of  $A$  is  $(1, 5)$ . Work out the values of  $p$  and  $q$ . [2 marks]

Answer  $p = \dots\dots$   $q = \dots\dots$

## E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The graph of  $y = x^2$  is translated by  $\begin{pmatrix} p \\ q \end{pmatrix}$ , then reflected in the  $y$ -axis. The result is  $y = x^2 + 10x + 28$ . Find  $p$  and  $q$ . [4 marks]

Answer  $p = \dots\dots$   $q = \dots\dots$

### Extension hints

Cover these with paper. Uncover one at a time.

- E1**   **Hint 1**   Write the equation after each step.
- Hint 2**    $y = (x - p)^2 + q$ , then  $y = (x + p)^2 + q$ .
- Hint 3**    $x^2 + 10x + 28 = (x + 5)^2 + 3$ .

### How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the image of a point on a transformed graph.   ☐ ☐ ☐

I can write the equation of a translated or reflected graph.   ☐ ☐ ☐

I can describe fully the transformation between two graphs.   ☐ ☐ ☐