

Vectors in geometry

About 50 minutes No calculator 42 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/gcse-vectors-in-geometry/

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** From chapter 19 [2 marks]
- A1** $\mathbf{a} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$. Work out $2\mathbf{a} + \mathbf{b}$.

Answer

The probability that it rains on Monday is 0.3. The probability that it rains on Tuesday is 0.8, independently. Find the probability that it rains on exactly one of the two days.

Answer

- A3** From chapter 3 [2 marks] **A4** From chapter 10 [2 marks]
- A3** A pie chart shows how 45 people travel to work. The sector for walking has an angle of 32° . Twice as many people cycle as walk. Everyone else drives. How many people drive?
- A4** Find the gradient and the y -intercept of the line $-5x - 2y = -4$.

Answer

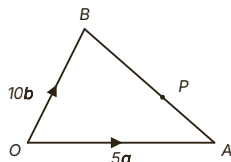
Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

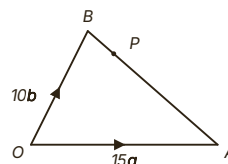
EXAMPLE 1

In triangle OAB , $\overrightarrow{OA} = 5\mathbf{a}$ and $\overrightarrow{OB} = 10\mathbf{b}$. P is on AB with $AP : PB = 2 : 3$. Write $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



Hint Find $\overrightarrow{AB}$ first. AP is $\frac{2}{5}$ of AB . Then go from O to A and on to P .

B1 In triangle OAB , $\overrightarrow{OA} = 15\mathbf{a}$ and $\overrightarrow{OB} = 10\mathbf{b}$. P is on AB with $AP : PB = 4 : 1$. Write $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3 marks]



Answer

EXAMPLE 2

$\overrightarrow{OA} = 4\mathbf{a} + \mathbf{b}$, $\overrightarrow{OB} = 6\mathbf{a} + 5\mathbf{b}$ and $\overrightarrow{OC} = 9\mathbf{a} + 11\mathbf{b}$. Show that A , B and C lie on a straight line.

Hint Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. If one is a multiple of the other, they are parallel, and they share the point B .

B2 $\overrightarrow{OP} = 3\mathbf{a} - 2\mathbf{b}$, $\overrightarrow{OQ} = \mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OR} = -4\mathbf{a} + 19\mathbf{b}$. Show that P , Q and R lie on a straight line, and write down the ratio $PQ : QR$. [3 marks]

C Practice

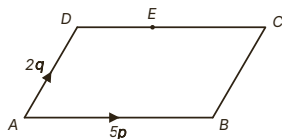
C1 $\overrightarrow{OA} = 4\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{OB} = 6\mathbf{a} + 8\mathbf{b}$. M is the midpoint of AB . Write $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

Answer

C2 $\overrightarrow{OA} = 3\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{a} + 4\mathbf{b}$. B is the midpoint of AC . Write $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

Answer

- C3** $ABCD$ is a parallelogram. [3 marks]
 $\overrightarrow{AB} = 5\mathbf{p}$ and $\overrightarrow{AD} = 2\mathbf{q}$. E is the point on DC with $DE : EC = 2 : 3$. Write $\overrightarrow{BE}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.



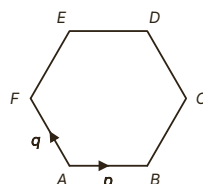
Answer

- C4** $\mathbf{a}$ and $\mathbf{b}$ are not parallel. [2 marks]
 $\overrightarrow{PQ} = 3\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{RS} = 12\mathbf{a} + k\mathbf{b}$. PQ is parallel to RS . Work out the value of k .

Answer

- C5** $\overrightarrow{OA} = 6\mathbf{a}$, $\overrightarrow{OB} = 4\mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OC} = 4\mathbf{b}$. Show that $OABC$ is a trapezium, and write down the ratio $CB : OA$. [3 marks]

- C6** $ABCDEF$ is a regular hexagon. [3 marks]
 $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AF} = \mathbf{q}$. Write $\overrightarrow{BD}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.



Answer

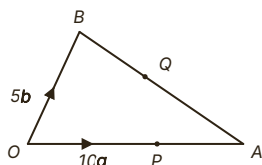
D Exam-style questions

- D1** In triangle OAB , $\overrightarrow{OA} = 9\mathbf{a}$ and $\overrightarrow{OB} = 9\mathbf{b}$. P is on AB with $AP : PB = 4 : 5$. Lily says, " $\overrightarrow{OP} = \frac{4}{9}\overrightarrow{OA} + \frac{5}{9}\overrightarrow{OB} = 4\mathbf{a} + 5\mathbf{b}$." [3 marks]

Show that Lily is wrong, and work out the correct $\overrightarrow{OP}$.

Answer

- D2** In triangle OAB , $\overrightarrow{OA} = 10\mathbf{a}$ and $\overrightarrow{OB} = 5\mathbf{b}$. P is on OA with $OP : PA = 3 : 2$. Q is on AB with $AQ : QB = 3 : 2$. R is the point on OB extended with $\overrightarrow{OR} = 9\mathbf{b}$.



- (a) Write $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1 mark]

Answer

(b) Write $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[2 marks]

Answer

(c) Prove that P , Q and R lie on a straight line. You must show your working.

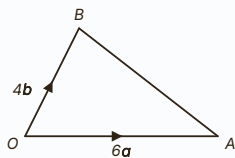
[3 marks]

Total for D2: 6 marks

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 In triangle OAB , $\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are not parallel. A line from O in the direction of $\mathbf{a} + \mathbf{b}$ meets AB at P . Work out the ratio $AP : PB$. [4 marks]



Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Let $\overrightarrow{AP} = t\overrightarrow{AB}$ and write $\overrightarrow{OP}$ in terms of t , $\mathbf{a}$ and $\mathbf{b}$.
Hint 2 $\overrightarrow{OP} = (6 - 6t)\mathbf{a} + 4t\mathbf{b}$. It is a multiple of $\mathbf{a} + \mathbf{b}$, so the two coefficients are equal.
Hint 3 $6 - 6t = 4t$, so $t = \frac{3}{5}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find a point that divides a line in a given ratio. ☐ ☐ ☐

I can prove that three points lie on a straight line. ☐ ☐ ☐

I can use vectors to show that lines in a shape are parallel. ☐ ☐ ☐