



Distance-time and velocity-time graphs, areas under curves, circles, other graphs and transformations

Grades 4–9 About 90 minutes Calculator allowed 96 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Work out the gradient of the straight line through (1, 3) and (5, 11).

[2 marks]

Answer

A2 A trapezium has parallel sides of 6 cm and 10 cm. The distance between them is 4 cm. Work out its area.

[2 marks]

Answer

A3 A train travels 252 km in 3.6 hours. Work out its average speed.

[1 mark]

Answer

A4 $f(x) = x^2 - 3$. Work out $f(4)$.

[1 mark]

Answer

A5 Write down the gradient of a line perpendicular to $y = 2x + 1$.

[1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A car starts from rest and speeds up steadily to 12 m/s in 4 seconds. It travels at 12 m/s for 6 seconds, then slows down steadily and stops 5 seconds later. Sketch the velocity-time graph and work out the total distance travelled.

Hint The distance is the area under a velocity-time graph. Split it into two triangles and a rectangle.

B1 [4 marks]

A cyclist starts from rest and speeds up steadily to 8 m/s in 5 seconds. She travels at 8 m/s for 10 seconds, then slows down steadily and stops 3 seconds later.

Work out her acceleration in the first 5 seconds, and the total distance she travels.

Answer m/s² m

EXAMPLE 2

Use 4 strips of equal width to estimate the area under the curve $y = x^2 + 1$ between $x = 0$ and $x = 4$.

Hint Work out y at $x = 0, 1, 2, 3, 4$. Each strip is a trapezium of width 1.

B2 [3 marks]

Use 3 strips of equal width to estimate the area under the curve $y = 10 - x^2$ between $x = 0$ and $x = 3$.

Answer

EXAMPLE 3

Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Hint The tangent is perpendicular to the radius. Find the gradient of the radius first.

B3 Find the equation of the tangent [3 marks]
to the circle $x^2 + y^2 = 52$ at the
point $(4, 6)$. Give your answer in
the form $ax + by = c$.

Answer

EXAMPLE 4

The graph of $y = f(x)$ has a turning point at $(2, -5)$. Write down the turning point of $y = f(x + 3)$ and of $y = -f(x)$.

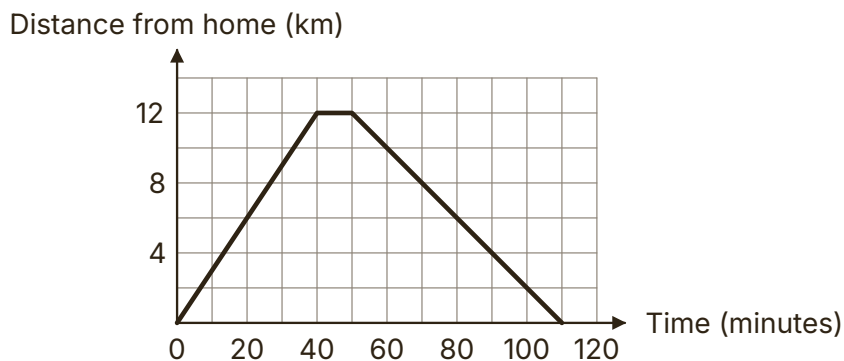
Hint $f(x + 3)$ moves the graph 3 to the left. $-f(x)$ reflects it in the x -axis.

B4 The graph of $y = f(x)$ has a [2 marks]
turning point at $(-4, 3)$. Write
down the turning point of
 $y = f(x) - 6$ and of $y = f(-x)$.

Answer (.....,)

Each question asks for something different. Show your working.

C1 Leah cycles from home to a park, rests, and then cycles home. The graph shows her journey.



(a) Work out her speed on the way to the park, in km/h.

[2 marks]

Answer

(b) For how many minutes does she rest?

[1 mark]

Answer

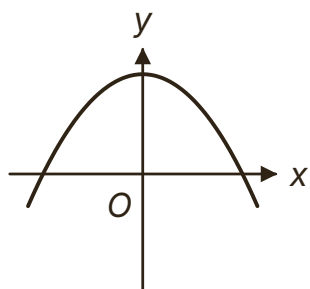
(c) Work out her speed on the way home, in km/h.

[2 marks]

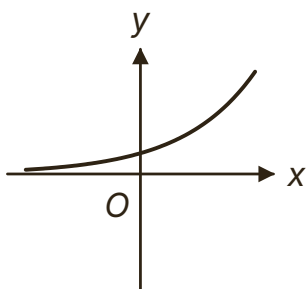
Answer

Total for C1: 5 marks

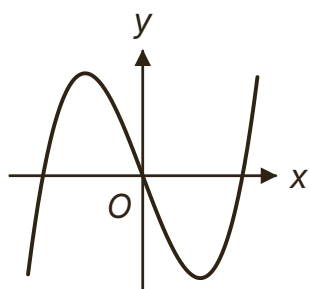
C2 Here are four sketch graphs. Match each equation to its graph.



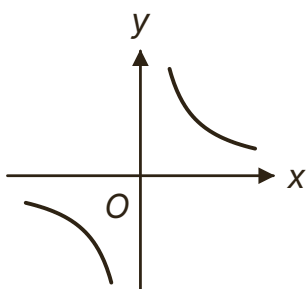
A



B



C



D

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(a) $y = x^3 - 4x$

[1 mark]

Answer

(b) $y = 4 - x^2$

[1 mark]

Answer

(c) $y = \frac{3}{x}$

[1 mark]

Answer

(d) $y = 2^x$

[1 mark]

Answer

Total for C2: 4 marks

C3 A circle has equation $x^2 + y^2 = 72$.

(a) Write down its radius in the form $a\sqrt{2}$.

[2 marks]

Answer

(b) Is the point (5, 7) inside the circle, on it, or outside it? Give a reason.

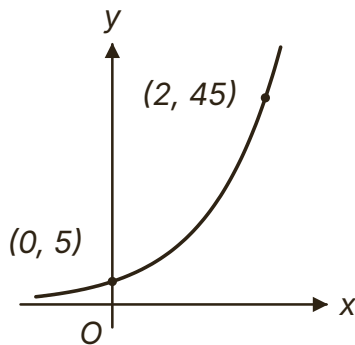
[2 marks]

Answer

Total for C3: 4 marks

- C4** The sketch shows the curve $y = ab^x$, where $b > 0$. The curve passes through (0, 5) and (2, 45). [3 marks]

Find the value of a and the value of b .



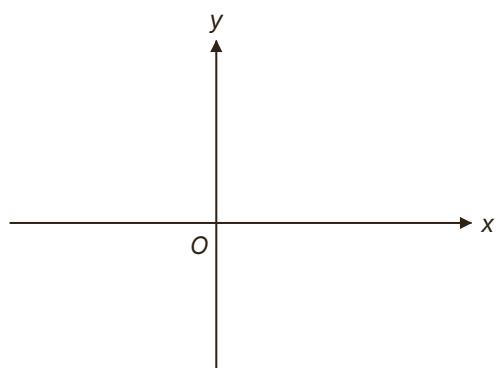
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Answer $a = \dots\dots$ $b = \dots\dots$

- C5** Describe fully the single transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 3)^2 + 2$. [2 marks]

Answer

- C6** Sketch the graph of $y = (x - 1)(x + 2)(x - 3)$. Show the coordinates of the points where it crosses the axes. [3 marks]



- C7** The table gives the velocity of a car at times during the first 6 seconds of a journey. Its velocity-time graph is a smooth curve whose gradient keeps falling. [3 marks]

Time (s)	0	2	4	6
Velocity (m/s)	0	5	8	9

- (a) Use 3 strips of equal width to estimate the distance the car travels in the first 6 seconds. [3 marks]

Answer

- (b) Is your answer an overestimate or an underestimate? Give a reason. [1 mark]

Answer

Total for C7: 4 marks

- C8** A circle has its centre at the origin and passes through the point $(-5, 12)$. Write down its equation. [2 marks]

Answer

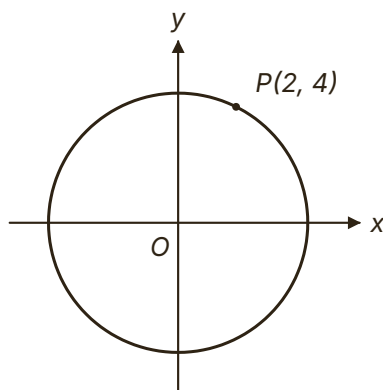
Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Kai says, "The graph of $y = f(x + 4)$ is the graph of $y = f(x)$ moved 4 units to the right." [2 marks]

Kai is wrong. Explain why, and describe the transformation fully.

Answer

- D2** The diagram shows the circle $x^2 + y^2 = 20$ and the point $P(2, 4)$ on it.



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- (a) Find the equation of the tangent to the circle at P . Give your answer in the form $ax + by = c$. [3 marks]

Answer

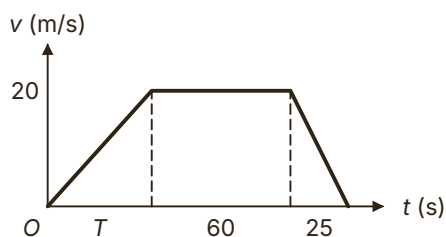
- (b) The tangent meets the x -axis at A and the y -axis at B . Work out the area of triangle OAB . [2 marks]

Answer

Total for D2: 5 marks

- D3** A train starts from rest and speeds up steadily to 20 m/s in T seconds. It travels at 20 m/s for 60 seconds, then slows down steadily and stops 25 seconds later. The train travels 1800 m altogether. [4 marks]

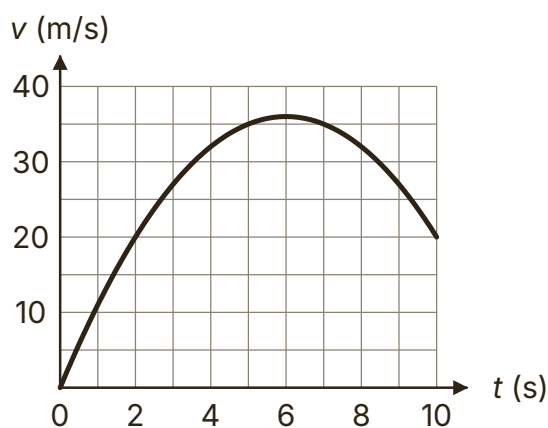
Work out the value of T . You must show your working.



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Answer

- D4** The graph shows the velocity, v m/s, of a model rocket car t seconds after it starts, where $v = 12t - t^2$ for $0 \leq t \leq 10$.



- (a) Use 5 strips of equal width to estimate the distance the car travels in the first 10 seconds. [3 marks]

Answer

(b) Is your answer to part (a) an overestimate or an underestimate? Give a reason. [1 mark]

Answer

(c) By drawing a tangent, estimate the acceleration of the car when $t = 4$. [2 marks]

Answer

Total for D4: 6 marks

D5 The value of a van, £ V , is modelled by $V = 18\,000 \times 0.85^t$, where t is its age in years.

(a) Work out the value of the van when it is 3 years old. [2 marks]

Answer

(b) After how many complete years is the value first less than £8000? [2 marks]

Answer

Total for D5: 4 marks

D6 Solve the simultaneous equations [4 marks]

$$y = 2x \quad x^2 + y^2 = 45$$

You must show your working.

Answer (.....,)

(.....,)

- D7** The graph of $y = x^2$ is reflected in the x -axis. The new graph is then translated by $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. [3 marks]
- Show that the final graph has equation $y = -x^2 + 6x - 5$.

-
- D8** The curve $y = \frac{k}{x}$ passes through the point $(4, -3)$.

(a) Find the value of k . [1 mark]

Answer

(b) Explain why the curve never meets the line $y = x$. [2 marks]

Answer

Total for D8: 3 marks

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- D9** A toy car moves in a straight line. The distance, d metres, it has travelled after t seconds is $d = 2t^2$.

(a) Work out the average speed of the car between $t = 1$ and $t = 4$. [2 marks]

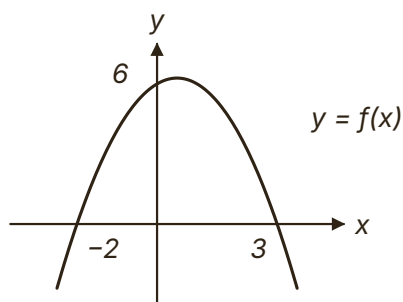
Answer

(b) Estimate the speed of the car at $t = 3$ by working out its average speed between $t = 2.9$ and $t = 3.1$. [2 marks]

Answer

Total for D9: 4 marks

- D10** The sketch shows the graph of $y = f(x)$. It crosses the x -axis at $x = -2$ and $x = 3$, and the y -axis at $y = 6$.



Not drawn accurately

- (a) Write down the coordinates of the points where the graph of $y = f(x - 1)$ crosses the x -axis. [2 marks]

Answer (.....,)

- (b) Write down the coordinates of the point where the graph of $y = -f(x)$ crosses the y -axis. [1 mark]

Answer (.....,)

Total for D10: 3 marks

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** Two tangents to the circle $x^2 + y^2 = 25$ have gradient $\frac{3}{4}$. Find their equations. [3 marks]

Answer

- E2** The graph of $y = 2^x$ is translated by $\begin{pmatrix} p \\ q \end{pmatrix}$. The new graph passes through (2, 5) and (4, 11). [3 marks]
Find the value of p and the value of q .

Answer $p = \dots\dots$ $q = \dots\dots$

- E3** P is a point on the circle $x^2 + y^2 = 50$. The tangent to the circle at P passes through the point (10, 0). [3 marks]
Find the two possible positions of P .

Answer $(\dots\dots, \dots\dots)$ $(\dots\dots, \dots\dots)$

- E4** Find where the curve $y = \frac{1}{x-2} + 3$ crosses each axis, and write down the equations of its two asymptotes. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can work out a speed from a distance-time graph, and an acceleration and a distance from a velocity-time graph.	☐	☐	☐
I can estimate the area under a curve with trapezia, say what it means, and say whether the estimate is too big or too small.	☐	☐	☐
I can use the equation $x^2 + y^2 = r^2$ and find the tangent to a circle at a given point.	☐	☐	☐
I can recognise cubic, reciprocal and exponential graphs from their equations.	☐	☐	☐
I can translate and reflect the graph of $y = f(x)$ and say where its points move to.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1**
- Hint 1** A tangent is perpendicular to the radius, so find where the radius has gradient $-\frac{4}{3}$.
- Hint 2** The radius has gradient $-\frac{4}{3}$ at $(-3, 4)$ and at $(3, -4)$.
- Hint 3** The tangent at $(-3, 4)$ is $y - 4 = \frac{3}{4}(x + 3)$, so $y = \frac{3}{4}x + \frac{25}{4}$. The tangent at $(3, -4)$ is $y = \frac{3}{4}x - \frac{25}{4}$.
-
- E2**
- Hint 1** Write the equation of the new graph using p and q .
- Hint 2** The new graph is $y = 2^{x-p} + q$, so $2^{2-p} + q = 5$ and $2^{4-p} + q = 11$.
- Hint 3** Subtracting, $2^{2-p}(4 - 1) = 6$, so $2^{2-p} = 2$. Then $p = 1$ and $q = 3$.
-
- E3**
- Hint 1** Call P the point (a, b) and write down the equation of the tangent at P .
- Hint 2** The tangent at (a, b) is $ax + by = 50$. It passes through $(10, 0)$, so $10a = 50$.
- Hint 3** $a = 5$, and $25 + b^2 = 50$ gives $b = 5$ or $b = -5$. So P is $(5, 5)$ or $(5, -5)$.
-
- E4**
- Hint 1** The curve is $y = \frac{1}{x}$ translated. Find the translation first.
- Hint 2** It is the translation by $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, so the asymptotes are $x = 2$ and $y = 3$.
- Hint 3** At $x = 0$, $y = -\frac{1}{2} + 3 = \frac{5}{2}$. At $y = 0$, $\frac{1}{x - 2} = -3$, so $x = \frac{5}{3}$.
-