



Congruent triangles, proof and right- angled trigonometry

Year 9 About 85 minutes Calculator allowed

89 marks

Name _____

Class _____

Date _____

A Before you start

Recall

You will need these in every question below.

A1 Two angles of a triangle are 53° and 71° . Work out the size of the third angle.

[1 mark]

Answer

A2 A right-angled triangle has shorter sides of 20 cm and 21 cm. Work out the length of its longest side.

[2 marks]

Answer

A3 Solve $\frac{x}{8} = 0.65$.

[1 mark]

Answer $x = \dots\dots$

A4 Solve $\frac{12}{x} = 0.75$.

[1 mark]

Answer $x = \dots\dots$

A5 Round 17.849 to 1 decimal place.

[1 mark]

Answer

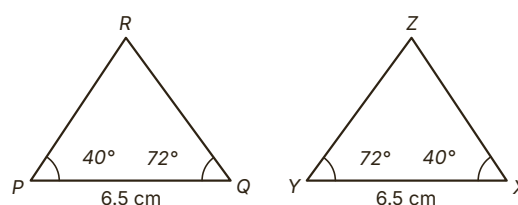
Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Triangles ABC and DEF have $AB = DE = 7$ cm, $BC = EF = 9$ cm, and angle $ABC =$ angle $DEF = 50^\circ$. Which condition shows that the triangles are congruent? Give a reason.

Hint Check where the angle sits. Angle B is between the sides AB and BC , so this is two sides and the angle between them.

B1 In the diagram, $PQ = XY = 6.5$ [2 marks]
cm, angle $P =$ angle $X = 40^\circ$
and angle $Q =$ angle $Y = 72^\circ$.
Which condition shows that the
triangles are congruent? Give a
reason.



Not drawn accurately

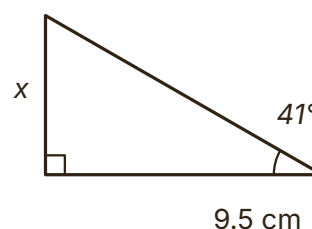
Answer condition

EXAMPLE 2

A right-angled triangle has a hypotenuse of 14 cm and an angle of 35° . Work out the length of the side opposite the 35° angle. Give your answer to 1 decimal place.

Hint Label the sides from the 35° angle: opposite, adjacent, hypotenuse. Opposite and hypotenuse means sine, so the side is $14 \sin 35^\circ$.

B2 Work out the length x . Give your [3 marks]
answer to 1 decimal place.



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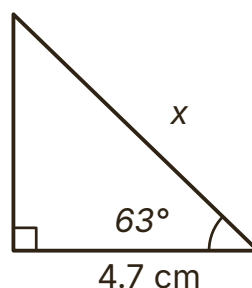
Answer

EXAMPLE 3

In triangle ABC , angle $B = 90^\circ$, angle $A = 28^\circ$ and $BC = 6$ cm. Work out the length of AC . Give your answer to 1 decimal place.

Hint BC is opposite the 28° angle and AC is the hypotenuse, so use sine. The unknown is on the bottom of the fraction, so you divide.

B3 Work out the length x . Give your answer to 1 decimal place. [3 marks]



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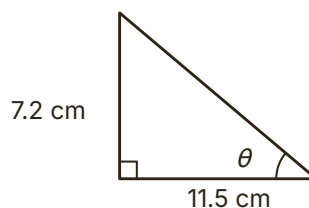
Answer

EXAMPLE 4

A right-angled triangle has a hypotenuse of 9 cm. The side opposite the angle θ is 5 cm. Work out the size of θ . Give your answer to 1 decimal place.

Hint Opposite and hypotenuse means sine: $\sin \theta = \frac{5}{9}$. To get the angle back from the ratio, use the inverse, $\sin^{-1}$.

B4 Work out the size of the angle θ . [3 marks]
Give your answer to 1 decimal place.

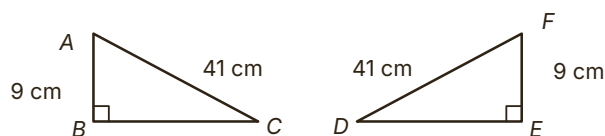


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Answer

Each question asks for something different. Show your working.

- C1** Triangles ABC and DEF each have a right angle. $AC = DF = 41$ cm and $AB = DE = 9$ cm.



Not drawn accurately

- (a) Which condition shows that the two triangles are congruent? [1 mark]

Answer

- (b) Work out the length of EF . [2 marks]

Answer

- (c) Work out the area of triangle DEF . [1 mark]

Answer

Total for C1: 4 marks

- C2** Triangle GHI has $GH = 11$ cm, $HI = 8$ cm and angle $HGI = 40^\circ$. Triangle JKL has $JK = 11$ cm, $KL = 8$ cm and angle $KJL = 40^\circ$. [2 marks]

Kim says, "The two triangles must be congruent." Is Kim right? Give a reason.

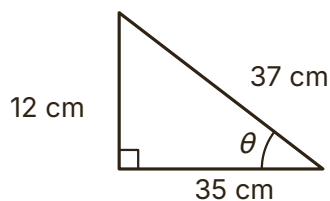
Answer Yes or No:

- C3** Triangles ABC and PQR are congruent. A matches P , B matches Q and C matches R . Angle $ABC = 74^\circ$ and angle $PRQ = 39^\circ$. [2 marks]

Work out the size of angle QPR .

Answer

- C4** The diagram shows a right-angled triangle with sides of 12 cm, 35 cm and 37 cm. Write each of these as a fraction.



Not drawn accurately

(a) $\sin \theta$

[1 mark]

Answer

(b) $\cos \theta$

[1 mark]

Answer

(c) $\tan \theta$

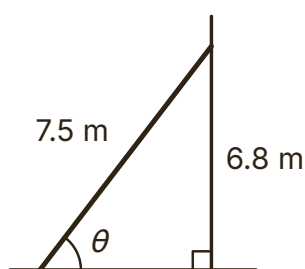
[1 mark]

Answer

Total for C4: 3 marks

- C5** A ladder 7.5 m long leans against a vertical wall. The top of the ladder is 6.8 m above the ground. [3 marks]

Work out the angle θ between the ladder and the ground. Give your answer to 1 decimal place.

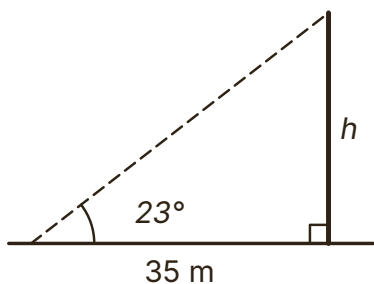


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Answer

- C6** Jo stands 35 m from the foot of a tree on level ground. The angle of elevation of the top of the tree from the ground where she stands is 23° . [3 marks]

Work out the height h of the tree. Give your answer to 1 decimal place.

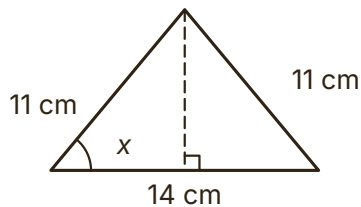


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Answer

- C7** An isosceles triangle has two sides of 11 cm and a base of 14 cm. The dashed line is its line of symmetry. [3 marks]

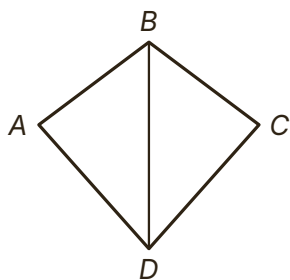
Work out the size of the angle x . Give your answer to 1 decimal place.



Not drawn accurately

Answer

C8 $ABCD$ is a kite with $AB = CB$ and $AD = CD$. The line BD is drawn.



(a) Prove that triangle ABD is congruent to triangle CBD .

[2 marks]

(b) Angle $BAD = 108^\circ$. Write down the size of angle BCD .

[1 mark]

Answer

Total for C8: 3 marks

D Test-style questions

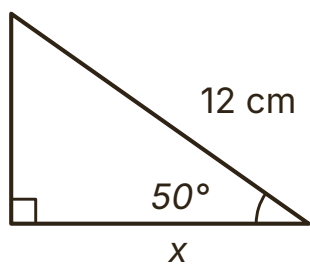
Stretch

Answer every question in the space given. Show your working: most of the marks are for method.

D1 Sam works out the length x . He writes, " $x = 12 \sin 50^\circ = 9.2$ cm."

[3 marks]

Explain the mistake Sam has made. Work out the correct value of x , to 1 decimal place.



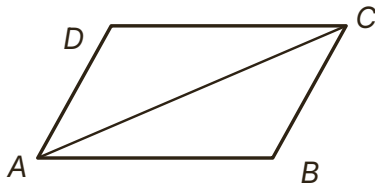
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Answer

D2 $ABCD$ is a parallelogram. The diagonal AC is drawn.

[3 marks]

Prove that triangle ABC is congruent to triangle CDA .



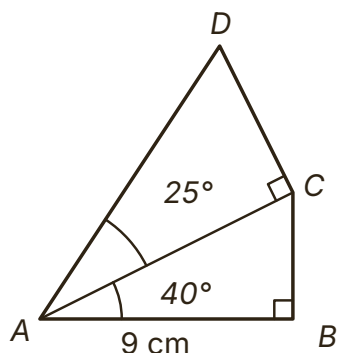
D3 Leah flies a kite. The string is 40 m long and is pulled straight. It makes an angle of 52° with the horizontal. Leah holds the end of the string 1.3 m above the ground. [3 marks]

Work out the height of the kite above the ground. Give your answer to 1 decimal place.

Answer

- D4** In the diagram, angle $ABC = 90^\circ$ and angle $ACD = 90^\circ$. $AB = 9$ cm, angle $BAC = 40^\circ$ [4 marks]
and angle $CAD = 25^\circ$.

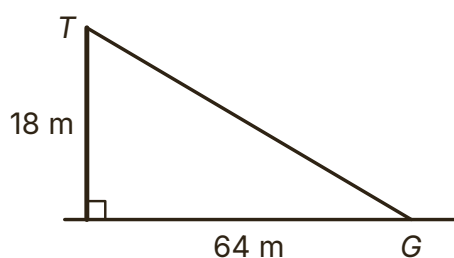
Work out the length of AD . Give your answer to 1 decimal place. You must show your working.



Not drawn accurately

Answer

- D5** A zip wire runs in a straight line from the top T of a tower 18 m tall to a point G on level ground 64 m from the foot of the tower.



Not drawn accurately

- (a)** Work out the angle between the zip wire and the ground. Give your answer to 1 [2 marks]
decimal place.

Answer

(b) Work out the length of the zip wire. Give your answer to 1 decimal place.

[2 marks]

Answer

(c) A safety rule says the angle between a zip wire and the ground must be no more than 12° . The tower stays the same. How much further from the tower must G be moved? Give your answer to 1 decimal place.

[2 marks]

Answer

Total for D5: 6 marks

D6 Triangles ABC and DEF are congruent. A matches D , B matches E and C matches F .
 $AB = (2y + 3)$ cm and $DE = (4y - 9)$ cm. $BC = 11$ cm and $AC = 13$ cm.

(a) Work out the value of y .

[2 marks]

Answer $y = \dots\dots$

(b) Work out the perimeter of triangle DEF .

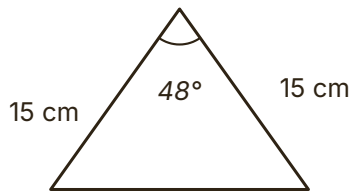
[2 marks]

Answer

Total for D6: 4 marks

- D7** An isosceles triangle has two sides of 15 cm. The angle between them is 48° . [4 marks]

Work out the area of the triangle. Give your answer to 1 decimal place. You must show your working.



Not drawn accurately

Answer

- D8** A boat sails 14 km due north from a harbour, then 9 km due east. [3 marks]

Work out the angle between due north and the straight line from the harbour to the boat. Give your answer to 1 decimal place.

Answer

- D9** In a right-angled triangle, the hypotenuse is 20 cm. One of the other angles is x , and $\tan x = 0.75$. [3 marks]

Work out the perimeter of the triangle. You must show your working.

Answer

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** A right-angled triangle has a perimeter of 60 cm. Its smallest angle is θ , and $\tan \theta = \frac{5}{12}$. Find the area of the triangle. [4 marks]

Answer

- E2** Triangle PQR has $PQ = PR$. M is the midpoint of QR . $QR = 30$ cm and the area of triangle PQR is 300 cm^2 . Find the perimeter of triangle PQR . [4 marks]

Answer

- E3** A ladder leans against a vertical wall and makes an angle of 62° with the level ground. The foot of the ladder is moved 1.5 m further from the wall. The ladder now makes an angle of 48° with the ground. Find the length of the ladder, to 1 decimal place. [4 marks]

Answer

E4 A right-angled triangle has a hypotenuse of 53 cm and an area of 630 cm^2 . Find the size of its smallest angle, to 1 decimal place. [4 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove that two triangles are congruent using SSS, SAS, ASA or RHS, and explain why two sides and a non-included angle are not enough.	☐	☐	☒
I can use sin, cos and tan to find a missing side or angle in a right-angled triangle.	☐	☐	☒

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	The two shorter sides are in the ratio 5 : 12. Call them $5k$ and $12k$.
	Hint 2	The hypotenuse is $\sqrt{(5k)^2 + (12k)^2} = 13k$, so the perimeter is $30k$.
	Hint 3	$30k = 60$, so $k = 2$. The shorter sides are 10 cm and 24 cm, and the area is $\frac{1}{2} \times 10 \times 24 = 120 \text{ cm}^2$.
E2	Hint 1	Triangles PQM and PRM are congruent (SSS), so the angles at M are equal. They add to 180° .
	Hint 2	So PM is perpendicular to QR and is the height of the triangle: $\frac{1}{2} \times 30 \times PM = 300$.
	Hint 3	$PM = 20$ cm and $QM = 15$ cm, so $PQ = \sqrt{15^2 + 20^2} = 25$ cm. The perimeter is $25 + 25 + 30 = 80$ cm.
E3	Hint 1	Call the length of the ladder L . Write the distance from the wall to the foot in each position.
	Hint 2	The distances are $L \cos 62^\circ$ and $L \cos 48^\circ$, and they differ by 1.5 m.
	Hint 3	$L(\cos 48^\circ - \cos 62^\circ) = 1.5$, so $L = \frac{1.5}{0.66913 - 0.46947} = 7.5$ m.
E4	Hint 1	Call the two shorter sides a and b . The area gives $ab = 1260$, and Pythagoras gives $a^2 + b^2 = 2809$.
	Hint 2	$(a + b)^2 = a^2 + b^2 + 2ab = 2809 + 2520 = 5329$, and $(a - b)^2 = 2809 - 2520 = 289$.
	Hint 3	$a + b = 73$ and $a - b = 17$, so the sides are 45 cm and 28 cm. The smallest angle is opposite the 28 cm side: $\tan^{-1} \left(\frac{28}{45} \right) = 31.9^\circ$.