



Standard form for very large and very small numbers, and upper and lower bounds

Years 8–9 About 80 minutes Calculator allowed
82 marks

Name _____

Class _____

Date _____

A Before you start

Recall

You will need these in every question below.

A1 Work out 4.7×1000 .

[1 mark]

Answer

A2 Work out $63 \div 10\,000$.

[1 mark]

Answer

A3 Write 10^{-2} as a decimal.

[1 mark]

Answer

A4 Round 8.374 to 2 decimal places.

[1 mark]

Answer

A5 Round 2846 to the nearest 100.

[1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Write 0.000 38 in standard form.

Hint Standard form is $A \times 10^n$ with $1 \leq A < 10$. Move the point until the number is between 1 and 10, and count the places. A small number has a negative power.

B1 Write 0.000 006 7 in standard form. [1 mark]

Answer

EXAMPLE 2

Work out $(4 \times 10^6) \times (7 \times 10^{-2})$. Give your answer in standard form.

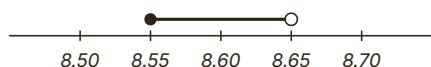
Hint Multiply the front numbers, then the powers of 10. If the front number is 10 or more, change it so it is between 1 and 10.

B2 Work out $(9 \times 10^8) \div (1.5 \times 10^3)$. [2 marks]
Give your answer in standard form.

Answer

EXAMPLE 3

A length, L , is 8.6 cm, correct to 1 decimal place. Write down the error interval for L .



Hint The length could be up to half of 0.1 either side of 8.6. The lower bound is included; the upper bound is not, because 8.65 would round up to 8.7.

B3 A mass, m , is 270 g, correct to the nearest 10 g. Write down the error interval for m . [2 marks]

Answer $\dots \leq m < \dots$

EXAMPLE 4

A rectangle is 12 cm by 7 cm. Both lengths are correct to the nearest cm. Work out the upper bound of its area.

Hint The greatest area comes from the greatest length and the greatest width. Each upper bound is half a centimetre more.

B4 A field is 60 m by 35 m. Both lengths are correct to the nearest 5 m. Work out the lower bound of its perimeter. [2 marks]

Answer

C Practice

Core

Each question asks for something different. Show your working.

C1 Answer each part.

(a) Write 9.04×10^5 as an ordinary number. [1 mark]

Answer

(b) Write 0.0507 in standard form. [1 mark]

Answer

(c) Write 46×10^3 in standard form. [1 mark]

Answer

Total for C1: 3 marks

C2 Write these numbers in order of size. Start with the smallest. [2 marks]

3.2×10^{-3} 0.0029 31×10^{-4} 3.05×10^{-3}

Answer

C3 Work out $(2.6 \times 10^5) + (4.7 \times 10^4)$. Give your answer in standard form.

[2 marks]

Answer

C4 One grain of rice has a mass of 2×10^{-2} g. How many grains of rice are there in a 1 kg bag? Give your answer in standard form.

[3 marks]

Answer

C5 A plank of wood is 2.4 m long, correct to the nearest 10 cm. Write down the lower bound of its length, in cm.

[2 marks]

Answer

C6 A bag holds 5.6 kg of flour. Kim takes out 1.3 kg. Both masses are correct to 1 decimal place. Work out the upper bound of the mass of flour left in the bag.

[3 marks]

Answer

C7 A runner covers 400 m, correct to the nearest 10 m, in 64 seconds, correct to the nearest second. Work out the greatest possible average speed of the runner, in m/s. Give your answer to 3 significant figures.

[3 marks]

Answer

- C8** A heart beats about 70 times each minute. Work out how many times it beats in a year of 365 days. Give your answer in standard form, correct to 2 significant figures. [3 marks]

Answer

- C9** A square has sides of 6.4 cm, correct to 1 decimal place. Work out the lower bound of its area. [2 marks]

Answer

D Test-style questions

Stretch

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Kay says, " 8×10^{-3} is bigger than 6×10^{-2} , because 8 is bigger than 6." [2 marks]

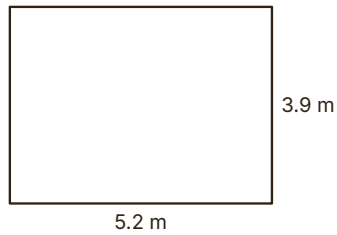
Explain why Kay is wrong. You must show your working.

Answer

- D2** A country has a population of 6.7×10^7 people. Its area is 2.4×10^5 km². Work out the number of people for each km² of the country. Give your answer to the nearest whole number. [3 marks]

Answer

- D3** The diagram shows the floor of a rectangular room. The lengths are correct to the nearest 0.1 m. Carpet costs £18 for each m^2 . Ali has £370 to spend on carpet.



- (a) Write down the upper bound of the length of the room. [1 mark]

Answer

- (b) Work out the upper bound of the area of the floor. [2 marks]

Answer

- (c) Show that Ali might not have enough money to carpet the room. [3 marks]

Answer

Total for D3: 6 marks

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- D4** An elephant has a mass of 6.3×10^3 kg. An ant has a mass of 4.5×10^{-6} kg.

- (a) How many times heavier is the elephant than the ant? Give your answer in standard form. [2 marks]

Answer

- (b) A nest has 2.4×10^5 ants. Work out the total mass of the ants in the nest, in kg. Give your answer as an ordinary number. [2 marks]

Answer

Total for D4: 4 marks

- D5 A car uses 32 litres of petrol, correct to the nearest litre, to travel 410 km, correct to the nearest 10 km. Work out the least possible number of kilometres the car travels for each litre. Give your answer to 3 significant figures. [3 marks]

Answer

- D6 A red blood cell is 7.5×10^{-6} m wide. A grain of pollen is 3×10^{-5} m wide.

- (a) How many times wider is the grain of pollen than the red blood cell? [2 marks]

Answer

- (b) Red blood cells are placed side by side in a line. How many whole cells fit along a line 1 cm long? [2 marks]

Answer

Total for D6: 4 marks

- D7 A water tank holds 450 litres, correct to the nearest 10 litres. Jo fills the tank using a jug that holds 1.5 litres, correct to 1 decimal place. Work out the greatest number of full jugs Jo could need to fill the tank. [3 marks]

Answer

- D8** A film file is 4.5×10^9 bytes. It downloads at 3×10^7 bytes each second. How many minutes does the film take to download? [3 marks]

Answer

- D9** The population of a city is 2.3×10^6 , correct to 2 significant figures. Write down the lower bound of the population. Give your answer in standard form. [2 marks]

Answer

- D10** A ribbon is 3.5 m long, correct to the nearest 0.1 m. Sam wants to cut it into 14 pieces, each 25 cm long, correct to the nearest cm. Show that he might not be able to cut 14 pieces. [3 marks]

Answer

E Challenge

UKMT level

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** A number N is multiplied by 400. The answer is 2.8×10^{-5} . Write N in standard form. [3 marks]

Answer

E2 A rectangle has a width of 6 cm, correct to the nearest cm. Its area is 54 cm^2 , correct to the nearest cm^2 . Work out the upper bound of its length. Give your answer to 3 significant figures. [4 marks]

Answer

E3 How many digits does the number $5^{12} \times 2^{15}$ have when it is written out in full? [3 marks]

Answer

E4 A number x rounds to 3, to the nearest whole number. A number y rounds to 30, to the nearest 10. Work out the difference between the upper bound and the lower bound of xy . [4 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write very large and very small numbers in standard form, and calculate with them.	☐	☐	☐
I can find the upper and lower bounds of a rounded measurement and use them in a calculation.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	Undo the multiplication: divide 2.8×10^{-5} by 400.
	Hint 2	Write 400 as 4×10^2 and divide the front numbers and the powers of 10 on their own.
	Hint 3	$2.8 \div 4 = 0.7$ and $10^{-5} \div 10^2 = 10^{-7}$, so $N = 0.7 \times 10^{-7} = 7 \times 10^{-8}$.

E2	Hint 1	The length is the area divided by the width.
	Hint 2	To make a division as big as possible, divide the biggest top number by the smallest bottom number.
	Hint 3	The upper bound of the area is 54.5 cm^2 and the lower bound of the width is 5.5 cm , so the length is at most $54.5 \div 5.5 = 9.909 \dots$, which is 9.91 cm .

E3	Hint 1	A 5 and a 2 multiplied together make 10. Pair them up.
	Hint 2	$5^{12} \times 2^{15} = 5^{12} \times 2^{12} \times 2^3 = 10^{12} \times 8$.
	Hint 3	8×10^{12} is an 8 followed by 12 zeros, which is 13 digits.

E4	Hint 1	Write down the bounds of x and of y first.
	Hint 2	x is between 2.5 and 3.5, and y is between 25 and 35. Both are positive, so the biggest product uses both upper bounds.
	Hint 3	The upper bound of xy is $3.5 \times 35 = 122.5$ and the lower bound is $2.5 \times 25 = 62.5$. The difference is $122.5 - 62.5 = 60$.