

Using congruent triangles to solve problems

Year 7 About 60 minutes No calculator 48 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/ks3-year-7-using-congruent-triangles-to-solve-problems/

Date _____

A Do Now

Quick questions to start.

A1 Needed today [1 mark] **A2** Needed today [1 mark]

Two straight lines cross. One of the angles is 38° . Work out the size of an angle next to it.

Answer

An isosceles triangle has an angle of 40° between its two equal sides. Work out one of the other angles.

Answer

A3 Last lesson [1 mark] **A4** From chapter 4 [1 mark]

Two triangles have two pairs of equal sides and the angles between them are equal. Which condition shows they are congruent?

Answer

A price goes up from £80 to £92. Work out the percentage increase.

Answer

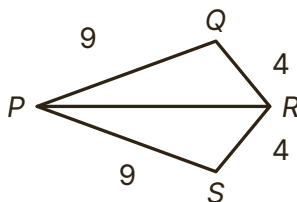
B Example, then your turn

Year 7

Your teacher works through each example with you. Then try the one beside it. All lengths are in cm.

EXAMPLE 1

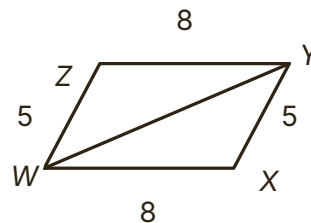
$PQRS$ is a kite with $PQ = PS$ and $QR = SR$. Show that angle $PQR =$ angle PSR .



Not drawn accurately

Hint Give a reason for each pair of equal parts. A side that is in both triangles is a common side.

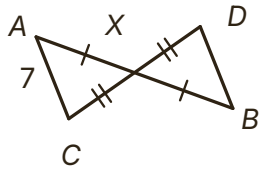
B1 $WXYZ$ has $WX = YZ$ and $XY = ZW$. Show that angle $WXY =$ angle YZW . [3 marks]



Not drawn accurately

EXAMPLE 2

AB and CD cross at X . X is the midpoint of AB and of CD . $AC = 7$ cm. Show that triangles AXC and BXD are congruent. Work out BD .

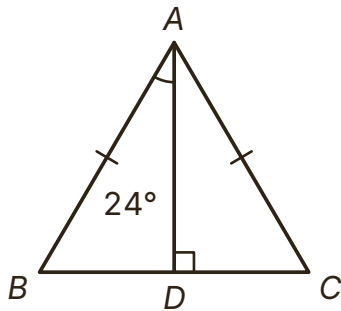


Not drawn accurately

Hint Where two straight lines cross, the opposite angles are equal.

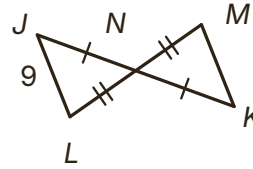
EXAMPLE 3

ABC is isosceles with $AB = AC$. AD is perpendicular to BC . Angle $BAD = 24^\circ$. Show that triangles ABD and ACD are congruent. Work out angle ACD .



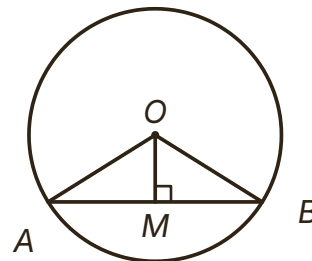
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Hint Both triangles have a right angle at D . Find the hypotenuse.

B2 JK and LM cross at N , the midpoint of both. $JL = 9$ cm and angle $LJN = 43^\circ$. Show that triangles JNL and KNM are congruent. Write down the length of KM and the size of angle MKN . [3 marks]

Not drawn accurately

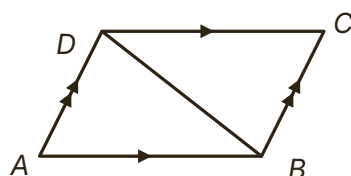
Answer

B3 O is the centre of a circle. AB is a chord of length 14 cm. OM is perpendicular to AB . Show that triangles OAM and OBM are congruent. Work out the length of AM . [3 marks]

Not drawn accurately

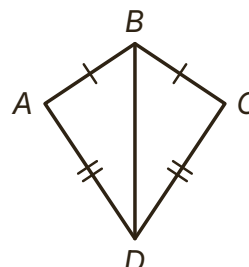
Answer

- C1** $ABCD$ is a parallelogram. Show that triangles ABD and CDB are congruent. Give a reason for each pair of equal parts. [3 marks]



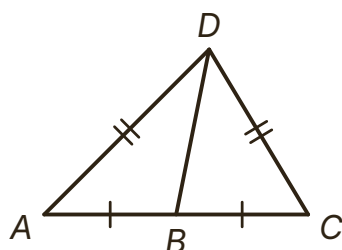
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- C2** In the kite $ABCD$, $AB = CB$ and $AD = CD$. Angle $ADB = (3x + 10)^\circ$ and angle $CDB = (5x - 14)^\circ$. Explain why the two angles are equal. Work out the value of x and the size of angle ADB . [3 marks]



Not drawn accurately

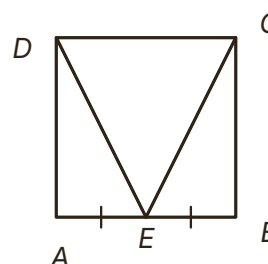
- C3** A , B and C lie on a straight line with $AB = BC$. $DA = DC$. Show that angle $DBA = 90^\circ$. [3 marks]



Not drawn accurately

Answer

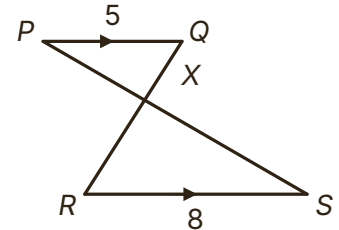
- C4** $ABCD$ is a square. E is the midpoint of AB . Show that $DE = CE$. [3 marks]



- D1** PQ is parallel to RS . PS and QR cross at X . Ben says, "Angle $QPX =$ angle RSX and angle $PQX =$ angle SRX (alternate angles), and angle $PXQ =$ angle SXR (vertically opposite). So triangles PQX and SRX are congruent."

[3 marks]

Is Ben correct? Give a reason for your answer.

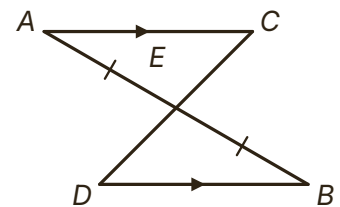


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- D2** AC is parallel to DB . AB and CD cross at E , and $AE = EB$.

- (a) Show that triangles AEC and BED are congruent. Give a reason for each pair of equal parts.

[3 marks]



Not drawn accurately

- (b) $AC = 6.4$ cm and $CE = 5.1$ cm. Work out the lengths of BD and CD .

[2 marks]

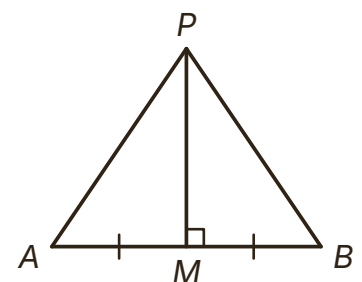
Answer

Total for D2: 5 marks

- D3** M is the midpoint of AB . The point P is on the line through M that is perpendicular to AB .

- (a) Show that triangles PAM and PBM are congruent.

[3 marks]



- (b) $AB = 10$ cm. The perimeter of triangle PAB is 36 cm. Work out the length of PA .

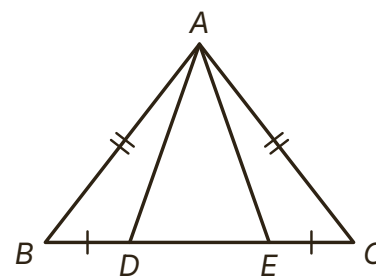
[2 marks]

Answer

Total for D3: 5 marks

D4 ABC is an isosceles triangle with $AB = AC$. D and E are on BC with $BD = CE$.

- (a) Show that triangles ABD and ACE are congruent. [3 marks]



- (b) Angle $ABC = 52^\circ$ and angle $BAD = 18^\circ$. [2 marks]
Work out the size of angle AEC .

Not drawn accurately

Answer

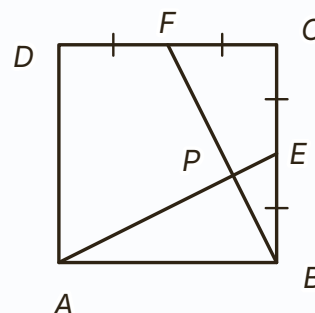
Total for D4: 5 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $ABCD$ is a square. E is the midpoint of BC and F is the midpoint of CD . The lines AE and BF cross at P . Prove that angle $APB = 90^\circ$. [5 marks]



Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Look at triangles ABE and BCF . Are they congruent?
Hint 2 They are congruent by SAS, so angle $BAE =$ angle CBF . Call this angle a .
Hint 3 Angle $ABP = 90 - a$. Now use the angles in triangle ABP .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can prove two triangles are congruent, with a reason for each pair of equal parts. ☐ ☐ ☐

I can use common sides, vertically opposite angles and alternate angles in a proof. ☐ ☐ ☐

I can use congruent triangles to find a missing side or angle. ☐ ☐ ☐

I know that three equal angles do not prove two triangles are congruent. ☐ ☐ ☐