



Hints online, answers free with an account:
mathswariz.uk/worksheets/ks3-year-8-solving-cubic-equations-by-drawing-graphs/

Solving cubic equations by drawing graphs

Year 8 About 75 minutes No calculator 46 marks

Date _____

A Do Now

Quick questions to start.

A1

Last lesson

[1 mark]

The lowest point of $y = x^2 - 6x + 1$ is $(3, -8)$. How many solutions does $x^2 - 6x + 1 = -8$ have?

Answer _____

A2

Needed today

[1 mark]

Work out the value of $x^3 - 5x$ when $x = -2$.

Answer _____

A3

From chapter 9

[1 mark]

Write 3.2×10^{-2} as an ordinary number.

Answer _____

A4

Year 7

[1 mark]

Solve $2x + 7 = 1$.

Answer _____

B Example, then your turn

Year 8

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Complete the table of values for $y = x^3 + 2x^2$. Use it to solve $x^3 + 2x^2 = 0$.

x	x^3	y
-3	-27	-9
-2		
-1	-1	
0		0

Hint Cube x (keep the sign), then add $2x^2$. The solutions are where $y = 0$.

B1

Complete the table of values [3 marks]

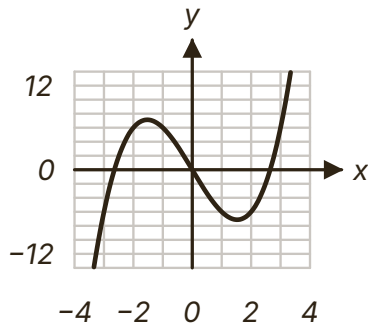
for $y = x^3 - x^2$. Use it to solve $x^3 - x^2 = 0$.

x	x^3	y
-1	-1	-2
0		
1	1	
2		

Answer $x = \dots\dots$ or $x = \dots\dots$

EXAMPLE 2

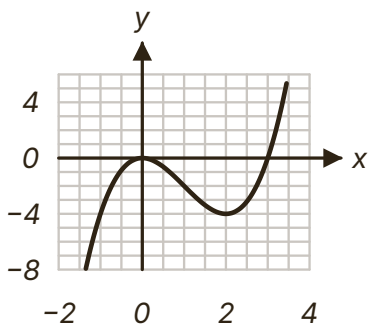
The graph of $y = x^3 - 7x$ is drawn. (a) Use it to solve $x^3 - 7x = 6$. (b) How many solutions do $x^3 - 7x = 10$ and $x^3 - 7x = 0$ have?



Hint Draw each line $y = 6$, $y = 10$, $y = 0$. Count where each one meets the curve.

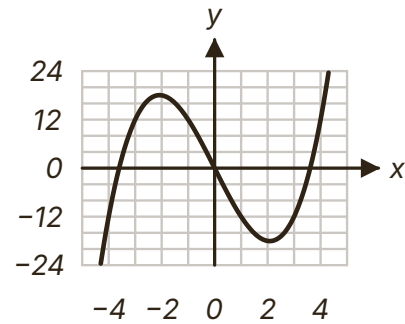
EXAMPLE 3

The graph of $y = x^3 - 3x^2$ is drawn. Use it to solve $x^3 - 3x^2 + 2 = 0$. Give your answers to 1 decimal place.



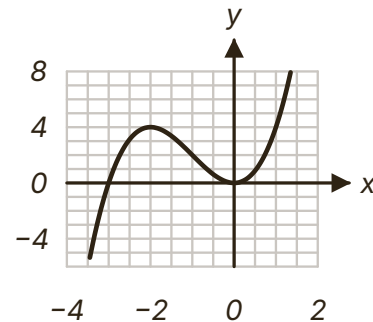
Hint Take 2 from both sides to get $x^3 - 3x^2 = -2$. Draw the line $y = -2$. Read off each crossing; two of them are between gridlines.

- B2** The graph of $y = x^3 - 13x$ is drawn. (a) Use it to solve $x^3 - 13x = 12$. (b) How many solutions does $x^3 - 13x = 20$ have? [3 marks]



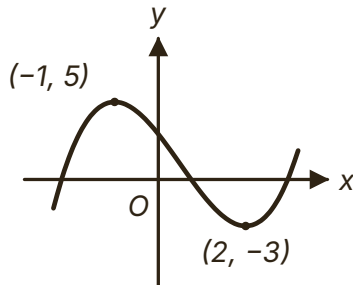
Answer

- B3** The graph of $y = x^3 + 3x^2$ is drawn. Draw a suitable line and use it to solve $x^3 + 3x^2 - 2 = 0$. Give your answers to 1 decimal place. [3 marks]



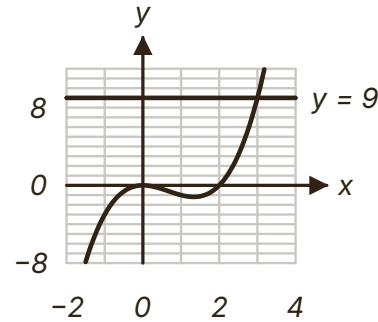
Answer $x = \dots\dots$, $x = \dots\dots$, $x = \dots\dots$

- C1** The sketch shows a cubic curve. [3 marks]
Its bends are at $(-1, 5)$ and $(2, -3)$. How many times does the line $y = k$ meet the curve when (a) $k = 0$, (b) $k = 5$, (c) $k = 8$?



Not drawn accurately

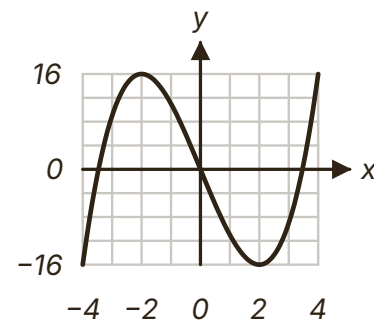
- C2** The curve $y = x^3 - 2x^2$ and the line $y = 9$ are drawn. Which equation do their crossing points solve? Write it in the form $x^3 + ax^2 + c = 0$, and write down its solution. [2 marks]



Answer

- C3** Ben says, "Some cubic equations have no solution." [2 marks]
Use the shape of a cubic graph to explain why Ben is wrong.

- C4** The graph of $y = x^3 - 12x$ is drawn. For which values of k does $x^3 - 12x = k$ have three solutions? [2 marks]



Answer

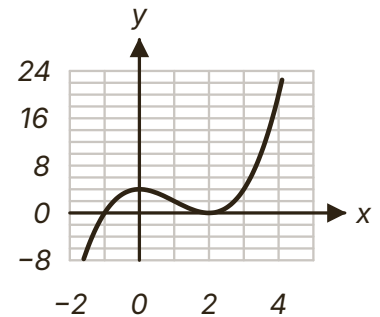
D1

Leo uses the graph of $y = x^3 - 3x^2 + 4$ to solve $x^3 - 3x^2 + 4 = 20$. He says, "It is a cubic, so it must have three solutions."

[4 marks]

Explain why Leo is wrong. Use the graph to solve the equation.

Answer $x = \dots\dots$



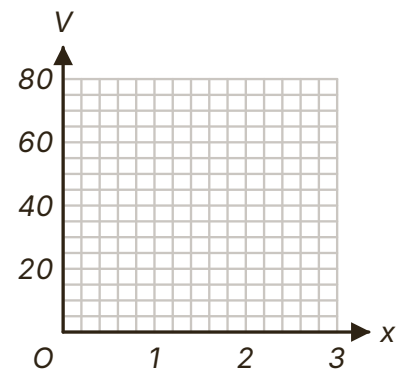
D2 A box has a base x cm by $2x$ cm. Its height is 1 cm more than x . Its volume is V cm³.

(a) Show that $V = 2x^3 + 2x^2$.

[1 mark]

(b) Work out V when $x = 1, 2$ and 3 . Plot the points on the grid and draw the graph of $V = 2x^3 + 2x^2$ through them and $(0, 0)$.

[2 marks]



(c) The volume of the box is 60 cm³. Use your graph to find x . Give your answer to 1 decimal place.

[2 marks]

Answer

Total for D2: 5 marks

D3 The graph of $y = x^3 - x^2 - 4x + 4$ is drawn on the grid.

(a) Use the graph to solve $x^3 - x^2 - 4x + 4 = 0$.

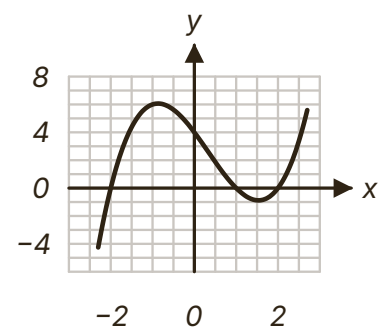
[2 marks]

Answer $x = \dots\dots$, $x = \dots\dots$, $x = \dots\dots$

(b) By drawing a suitable line, solve

[2 marks]

$x^3 - x^2 - 4x + 3 = 0$. Give your answers to 1 decimal place.



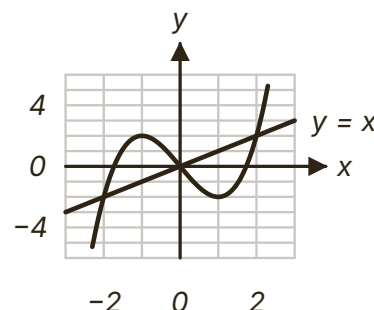
Answer $x = \dots\dots$, $x = \dots\dots$, $x = \dots\dots$

- (c) How many solutions does $x^3 - x^2 - 4x + 4 = -3$ have? Explain how you know [2 marks]
from the graph.

Total for D3: 6 marks

D4 The graph of $y = x^3 - 3x$ and the line $y = x$ are drawn on the grid.

- (a) Write down the coordinates of the three points where the line meets the curve. [2 marks]



Answer

- (b) How many solutions does $x^3 - 3x = 2$ have? [2 marks]
Explain how you know from the graph.

Total for D4: 4 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve $y = x^3 - 3x^2 - 9x$ has its two bends at $x = -1$ and $x = 3$. The equation $x^3 - 3x^2 - 9x + k = 0$ has exactly two solutions. Find the two possible values of k . [5 marks]

Answer $k = \dots\dots$ or $k = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Write it as $x^3 - 3x^2 - 9x = -k$. Exactly two solutions means the line $y = -k$ just touches a bend.
- Hint 2** Work out the y -value of each bend: put $x = -1$ and $x = 3$ into $x^3 - 3x^2 - 9x$.
- Hint 3** The bends are $(-1, 5)$ and $(3, -27)$. So $-k = 5$ or $-k = -27$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can complete a table of values for a cubic and read solutions where $y = 0$. ○ ○ ○
- I can draw a line $y = k$ on a cubic graph and read the solutions. ○ ○ ○
- I can say whether a cubic equation has three, two or one solution. ○ ○ ○