



Linear, special and quadratic sequences

Grades 4–9 About 80 minutes No calculator 87 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Write down the next two terms of the sequence 12, 21, 30, 39, ... [1 mark]

Answer _____

A2 Write down the term-to-term rule of the sequence 40, 34, 28, 22, ... [1 mark]

Answer _____

A3 Work out the value of $3n - 2$ when $n = 9$. [1 mark]

Answer _____

A4 Write down the first three square numbers that are greater than 50. [1 mark]

Answer _____

A5 The first term of a sequence is 4. The term-to-term rule is "multiply by 3". Work out the fourth term. [1 mark]

Answer _____

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Find the n th term of the sequence 2, 11, 20, 29, ...

Hint The terms go up by 9, so the n th term starts $9n$. Compare $9n$ with the sequence to find what to add or subtract.

B1 Find the n th term of the sequence 6, 14, 22, 30, ... [2 marks]

Answer

EXAMPLE 2

Is 97 a term of the sequence with n th term $3n + 4$?

Hint Set the n th term equal to 97 and solve. The term number has to be a whole number.

B2 Is 123 a term of the sequence with n th term $5n - 3$? Give a reason. [2 marks]

Answer

EXAMPLE 3

Find the n th term of the sequence 4, 9, 18, 31, 48, ...

Hint Find the first differences, then the second differences. Half the second difference is the number in front of n^2 . Take that away from each term and find the n th term of what is left.

B3 Find the n th term of the sequence 4, 12, 26, 46, 72, ... [3 marks]

Answer

EXAMPLE 4

Pattern 1 uses 6 tiles, pattern 2 uses 10 tiles and pattern 3 uses 14 tiles. The patterns carry on in the same way. How many tiles are in pattern 40?

Hint Each pattern uses 4 more tiles than the one before. Find the n th term first, then put $n = 40$ into it.

B4 Pattern 1 uses 7 matches, pattern 2 uses 12 matches and pattern 3 uses 17 matches. The patterns carry on in the same way. How many matches are in pattern 30? [2 marks]

Answer

C Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 Here is a sequence: 41, 37, 33, 29, ...

(a) Find the n th term.

[2 marks]

Answer

(b) Work out the first term of the sequence that is negative.

[2 marks]

Answer

Total for C1: 4 marks

C2 In a sequence, each term after the second is the sum of the two terms before it. The first term is 2 and the second term is 5. Work out the seventh term. [2 marks]

Answer

C3 Here is a sequence: 1, 3, 6, 10, 15, ... [2 marks]

Write down the next term, and the name of this sequence.

Answer

C4 The diagram shows the first three patterns in a sequence of dots. Pattern n has n rows of dots.



(a) How many dots are in pattern 4? [1 mark]

Answer

(b) Write down an expression for the number of dots in pattern n . [2 marks]

Answer

(c) Which pattern has 180 dots? [2 marks]

Answer

Total for C4: 5 marks

C5 Here is a geometric sequence: 81, 54, 36, 24, ... [2 marks]

Write down the common ratio and work out the next term.

Answer

C6 Find the n th term of the sequence 6, 16, 32, 54, 82, ...

[3 marks]

Answer

C7 The n th term of a sequence is $n^2 - 3n + 5$. Which term of the sequence is equal to 59? [3 marks]

Answer

C8 Sequence A has n th term $3n + 2$. Sequence B has n th term $62 - 3n$. For one value of n the two sequences have the same term. Find that value of n and the term. [3 marks]

Answer

C9 Find the n th term of the sequence 0.5, 0.8, 1.1, 1.4, ... [2 marks]

Answer

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Jo says, "The n th term of the sequence 7, 16, 25, 34, ... is $n + 9$, because you add 9 each time." [3 marks]

Explain why Jo is wrong, and write down the correct n th term.

Answer

- D2** Here is a sequence: 7, 13, 19, 25, ... [3 marks]
Is 200 a term of this sequence? You must show your working.

Answer

- D3** Here is a quadratic sequence: 3, 12, 25, 42, 63, ...
(a) Find the n th term. [3 marks]

Answer

- (b) Work out the 20th term. [2 marks]

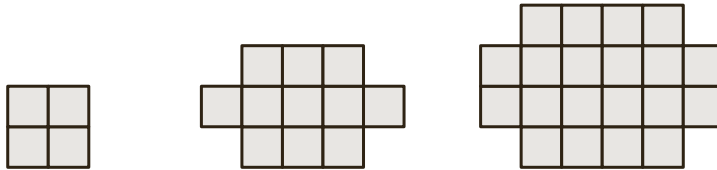
Answer

Total for D3: 5 marks

- D4** The diagram shows the first three patterns in a sequence made from square tiles. The patterns carry on in the same way. [6 marks]

Gemma has 300 tiles. She makes the largest pattern in the sequence that she can.

How many tiles does she have left over? You must show your working.



Pattern 1

Pattern 2

Pattern 3

Answer

- D5** In a sequence, each term after the second is the sum of the two terms before it. The third term is 15 and the fifth term is 41. [4 marks]

Find the first term and the second term.

Answer

- D6** Find the n th term of the sequence $\frac{3}{5}, \frac{5}{8}, \frac{7}{11}, \frac{9}{14}, \dots$ [3 marks]

Answer

D7 The n th term of a sequence is $85 - 6n$.

(a) Show that 31 is a term of the sequence.

[2 marks]

(b) How many terms of the sequence are positive?

[2 marks]

Answer

Total for D7: 4 marks

D8 The 3rd term of a linear sequence is 17 and the 7th term is 41.

[4 marks]

Find the n th term, and the first term of the sequence that is greater than 100.

Answer

D9 The n th term of a sequence is $an^2 + bn$. The first term is 1 and the second term is 8.

[3 marks]

Find the values of a and b , and work out the third term.

Answer

D10 The n th triangular number is $\frac{n(n+1)}{2}$.

[3 marks]

Show that the sum of any two triangular numbers next to each other in the sequence is a square number.

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 Find the n th term of the sequence 1, 7, 19, 37, 61, ...

[3 marks]

Answer

E2 The first 100 terms of the sequence with n th term $4n + 3$ are written down.
How many of them are square numbers?

[3 marks]

Answer

E3 In a sequence, each term after the second is the sum of the two terms before it. The first term is 2 and the sixth term is 46. [3 marks]
Find the second term.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find the n th term of a linear sequence and use it to test whether a number is a term.	☐	☐	☐
I can recognise square, cube, triangular and Fibonacci-type sequences and write a rule for a growing pattern.	☐	☐	☐
I can find the n th term of a quadratic sequence using second differences.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- | | | |
|-----------|---------------|--|
| E1 | Hint 1 | Work out the first differences and then the second differences. |
| | Hint 2 | The second difference is 6, so the n th term starts $3n^2$. Take $3n^2$ away from each term. |
| | Hint 3 | $1 - 3 = -2$, $7 - 12 = -5$, $19 - 27 = -8$: this is $1 - 3n$. So the n th term is $3n^2 - 3n + 1$. |
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- | | | |
|-----------|---------------|---|
| E2 | Hint 1 | Write down the first few terms and the first few square numbers. What do you notice when you divide them by 4? |
| | Hint 2 | Every term $4n + 3$ leaves a remainder of 3 when you divide it by 4. An even square, $(2k)^2 = 4k^2$, leaves remainder 0. |
| | Hint 3 | An odd square is $(2k + 1)^2 = 4k^2 + 4k + 1$, which leaves remainder 1. No square leaves remainder 3, so none of the terms is a square number: the answer is 0. |
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- | | | |
|-----------|---------------|---|
| E3 | Hint 1 | Call the second term b . Write the third, fourth, fifth and sixth terms in terms of b . |
| | Hint 2 | The terms are 2, b , $2 + b$, $2 + 2b$, $4 + 3b$, $6 + 5b$. |
| | Hint 3 | $6 + 5b = 46$, so $5b = 40$ and the second term is 8. |