The Argand diagram, modulus and argument

87 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-argand-modarg/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Write down the exact values, in radians, of $\tan^{-1}\sqrt{3}$ and $\tan^{-1}(-1)$.

Answer .....

A2 Needed today [2 marks]

Write 135° in radians, and $\frac{5\pi}{6}$ in degrees.

Answer .....

A3 Last lesson [2 marks]

$z = 3 - 2i$. Write down z^*, and find zz^*.

Answer .....

A4 From A-Level Maths [2 marks]

Find the distance between the points $(-1, 4)$ and $(5, -4)$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Plotting points, the conjugate and distances

EXAMPLE 1 · I DO

$z = -3 + 4i$. Plot z and z^* on an Argand diagram. Find $|z|$, and describe how the points z and z^* are related.

Hint The point is $(-3, 4)$. The modulus is its distance from O.

YOUR TURN · WE DO

B1 $w = 5 - 12i$. Write down $\mathrm{Re}(w)$, $\mathrm{Im}(w)$ and w^*. Find $|w|$, and describe in geometrical terms how the points w and w^* are related. [4 marks]

Answer .....

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EXAMPLE 2 · I DO

$z_1 = 1 + 3i$ and $z_2 = 4 + i$. Show z_1, z_2, $z_1 + z_2$ and $z_2 - z_1$ on an Argand diagram. Find $|z_2 - z_1|$ and explain what it measures.

Hint $z_1 + z_2$ is the fourth corner of the parallelogram with sides Oz_1 and Oz_2. $z_2 - z_1$ is the step from the point z_1 to the point z_2.

YOUR TURN · WE DO

B2 $z_1 = -2 + 5i$ and $z_2 = 4 - 3i$. Find $z_1 + z_2$ and the distance between the points z_1 and z_2. Find the complex number at the midpoint of z_1 and z_2. [5 marks]

Answer .....

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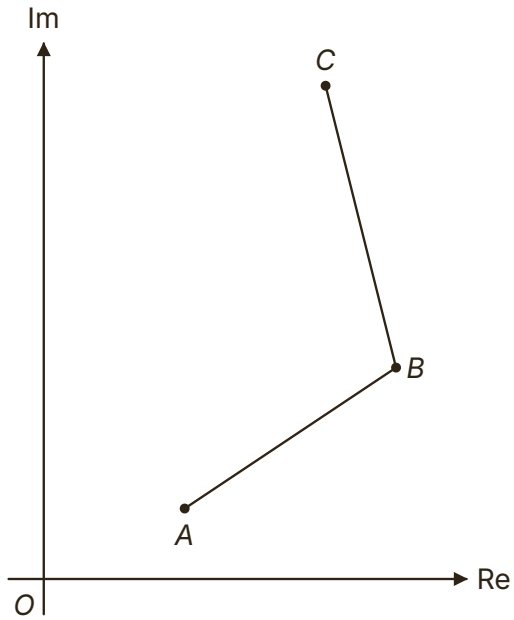
C1 The points A, B and C in the diagram represent $2 + i$, $5 + 3i$ and $4 + 7i$. $ABCD$ is a parallelogram. Find the complex number d represented by D, and the length of the diagonal BD. [4 marks]



Answer .....

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C2 The points A, B and C represent $1 + 2i$, $7 + 10i$ and $9 - 4i$. Show that triangle ABC is isosceles and right-angled. [5 marks]

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PART 2 Modulus and argument in every quadrant

EXAMPLE 3 · I DO

Find the exact modulus and argument of $2 + 2\sqrt{3}i$. Find the modulus and argument of $3 + 2i$, giving the argument to 3 significant figures.

Hint In the first quadrant, $\arg z = \tan^{-1}\dfrac{y}{x}$. Use exact values where you can.

YOUR TURN · WE DO

B3 Find the modulus and argument of (i) $3\sqrt{3} + 3i$, exactly (ii) $1 + 4i$, giving the argument to 3 significant figures. [4 marks]

Answer .....

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EXAMPLE 4 · I DO

Find the modulus and the argument, in $(-\pi, \pi]$, of (i) $-4+3i$, to 3 significant figures (ii) $-1-i$, exactly. Then give the argument of $-1-i$ in the interval $[0, 2\pi)$.

Hint Sketch the point first. Find the angle the line makes with the real axis, then use the sketch to give $\arg z$. For $[0, 2\pi)$, add 2π to a negative argument.

YOUR TURN · WE DO

B4 Find the modulus and the argument, in $(-\pi, \pi]$, of (i) $-\sqrt{3}+i$, exactly (ii) $-5-12i$, to 3 significant figures. [5 marks]

Answer .....

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C3 Find the modulus and the argument, in $(-\pi, \pi]$, of (i) $-6 + 6i$, exactly (ii) $2 - 5i$, giving the argument to 3 significant figures. [4 marks]

Answer .....

C4 Sam is asked for $\arg(-2 - 7i)$ and writes [4 marks]

$$\arg(-2 - 7i) = \tan^{-1}\frac{-7}{-2} = \tan^{-1}\frac{7}{2} = 1.29.$$

Explain Sam's mistake. Find $\arg(-2 - 7i)$ in $(-\pi, \pi]$, and in $[0, 2\pi)$, to 3 significant figures.

Answer .....

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C5 $z = 2 + i$ and $w = z^2$. Find $|z|$, $|w|$, $\arg z$ and $\arg w$, giving arguments to 4 significant figures. What do you notice? [5 marks]

Answer .....

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PART 3 Finding z from its modulus and one more fact

EXAMPLE 5 · I DO

$|z| = 10$, $\mathrm{Re}(z) = -6$ and $\arg z$ is negative. Find z, and find $\arg z$ to 3 significant figures.

Hint Use $|z|^2 = x^2 + y^2$ to find the imaginary part, then use the sign of $\arg z$ to choose.

YOUR TURN · WE DO

B5 $|z| = 13$, $\mathrm{Im}(z) = 5$ and $\mathrm{Re}(z) < 0$. Find z, and find $\arg z$ in the interval $[0, 2\pi)$, to 3 significant figures. [5 marks]

Answer .....

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C6 $|z| = 17$, $\mathrm{Re}(z) = 8$ and $\mathrm{Im}(z) < 0$. Find z, and find $\arg z$ in the interval $(-\pi, \pi]$, to 3 significant figures. [4 marks]

Answer .....

C7 The points A and B represent $2 + 6i$ and $8 + 2i$. The point C represents z_3, where $\arg z_3 = \dfrac{\pi}{4}$, and $CA = CB$. Find z_3. [6 marks]

Answer .....

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D1 Alex writes: "$|z_1 + z_2| = |z_1| + |z_2| = 10 + 13 = 23$."

$z_1 = 6 + 8i$ and $z_2 = 5 - 12i$.

(a) Show that Alex is wrong. [3 marks]

(b) Use an Argand diagram to explain why $|z_1 + z_2| \leq |z_1| + |z_2|$ for any complex numbers z_1 and z_2. State when the two sides are equal. [3 marks]

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(c) Find $|z_1 - z_2|$ exactly, and $\arg(z_1 - z_2)$ to 3 significant figures. [3 marks]

Answer .....

Total for D1: 9 marks

D2 In this question you must show detailed reasoning. The complex number z satisfies $\arg z = -\dfrac{3\pi}{4}$ and $|z + 4| = 2\sqrt{5}$. Find z exactly. [7 marks]

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Two adjacent vertices of a square are the points $1 + 2i$ and $4 + 6i$. Find, as complex numbers, all the possible positions of the other two vertices, and the centre of each possible square. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The step from one vertex to the next is the complex number $(4 + 6i) - (1 + 2i)$.

Hint 2 A step at right angles to $a + bi$, with the same length, is $-b + ai$ or $b - ai$.

Hint 3 The square can be on either side of the given edge. The centre is the midpoint of a diagonal.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can plot z, z^*, $z_1 + z_2$ and $z_1 - z_2$ on an Argand diagram and describe them geometrically.	○ ○ ○
I can find $\lvert z \rvert$, and $\lvert z_1 - z_2 \rvert$ as the distance between two points.	○ ○ ○
I can find $\arg z$ in any quadrant, in $(-\pi, \pi]$ or in $[0, 2\pi)$, using a sketch.	○ ○ ○
I can find z from its modulus and one more fact about it.	○ ○ ○

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