Complex numbers: arithmetic and conjugates

86 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-complex-arithmetic/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Expand and simplify $(3+2\sqrt{5})(1-\sqrt{5})$.

Answer .....

A2 Needed today [2 marks]

Write $\dfrac{4}{3-\sqrt{2}}$ with a rational denominator.

Answer .....

A3 Needed today [2 marks]

Find the discriminant of $x^2-6x+13$. How many real roots does $x^2-6x+13=0$ have?

Answer .....

A4 From A-Level Maths [2 marks]

Solve $3x-2y=5$ and $2x+y=8$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Adding, subtracting and multiplying

EXAMPLE 1 · I DO

$z = 2 + 3i$ and $w = 4 - i$. Find $z - w$ and zw.

Hint Multiply out as you would two brackets, then use $i^2 = -1$.

EXAMPLE 2 · I DO

Simplify i^7 and $(1 - i)^2$.

Hint $i^4 = 1$, so take out as many i^4 as you can.

YOUR TURN · WE DO

B1 Find, in the form $a + bi$, (i) $(5 - 2i)(3 + 4i)$ (ii) $(1 + 2i)^2$. [4 marks]

Answer .....

YOUR TURN · WE DO

B2 Simplify (i) i^{23} (ii) $(1 + i)^4$. [4 marks]

Answer .....

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C1 $z = 2 + 5i$ and $w = 1 - 3i$. Find, in the form $a + bi$, (i) $z + w$ (ii) zw. [3 marks]

Answer .....

C2 Simplify (i) i^{14} (ii) $(2 - i)^3$, giving (ii) in the form $a + bi$. [3 marks]

Answer .....

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C3 Expand and simplify $(2+3i)(2-3i)(1+i)$. [3 marks]

Answer .....

PART 2 Dividing by a complex number

EXAMPLE 3 · I DO

Write $\dfrac{3+4i}{1-2i}$ in the form $a+bi$.

Hint Multiply the top and the bottom by $1+2i$, the conjugate of the bottom.

YOUR TURN · WE DO

B3 Write $\dfrac{7-i}{2+i}$ in the form $a+bi$. [4 marks]

Answer .....

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C4 Write $\dfrac{10}{3+i}$ in the form $a+bi$. [3 marks]

Answer .....

C5 Sam works out $\dfrac{2+i}{1+i}$ like this:

$$\frac{(2+i)(1+i)}{(1+i)(1+i)} = \frac{1+3i}{2i}.$$

Explain Sam's mistake, and write $\dfrac{2+i}{1+i}$ in the form $a+bi$. [4 marks]

Answer .....

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C6 Write $\dfrac{2+3i}{1-i} - \dfrac{1+2i}{1+i}$ in the form $a+bi$. [5 marks]

Answer .....

PART 3 Equating real and imaginary parts

EXAMPLE 4 · I DO

Solve $z + 2z^* = 9 - 4i$.

Hint Write $z = x + iy$, so $z^* = x - iy$. Then the real parts are equal and the imaginary parts are equal.

YOUR TURN · WE DO

B4 Solve $3z - 2iz^* = 5$. [5 marks]

Answer .....

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EXAMPLE 5 · I DO

$w = a + i$, where a is real. $w^2 + 2w^*$ is purely imaginary. Find the exact values of a.

Hint Write $w^2 + 2w^*$ in the form $X + Yi$. Purely imaginary means $X = 0$.

YOUR TURN · WE DO

B5 In this question you must show detailed reasoning. $w = a + 2i$, where a is real. $w^2 + 4w^*$ is purely imaginary. Find the exact values of a. [5 marks]

Answer .....

PRACTICE · YOU DO

C7 Solve $2z - 3z^* = 1 + 10i$. [3 marks]

Answer .....

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C8 Find the real numbers x and y for which $(x + iy)(2 - 3i) = 4 + 7i$. [4 marks]

Answer .....

C9 $z = 1 + 2i$. Find z^3 in the form $a + bi$. Hence find the real numbers p and q for which $z^3 + pz + q = 0$. [6 marks]

Answer .....

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D Exam-style questions

OCR exam style

D1 Alex writes: "$(a+bi)^2 = a^2 - b^2 + 2abi$, so z^2 is real only when $b = 0$."

$z = a + bi$, where a and b are real.

(a) Explain why Alex is wrong. [2 marks]

(b) Now $b \neq 0$, and $z^2 + 2z^*$ is real. Show that $a = 1$. [3 marks]

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(c) Given also that $z^2 + 2z^* = -6$, find the two possible values of z. [3 marks]

Answer .....

Total for D1: 8 marks

D2 Solve the simultaneous equations [6 marks]

$$z + 2w = 4 - i, \qquad iz - w = -2 + 3i,$$

giving z and w in the form $a + bi$.

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find every complex number z for which $z^2 = z^*$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $z = x + iy$ on both sides, and equate the real parts and the imaginary parts.

Hint 2 The imaginary parts give $2xy = -y$. Do not divide by y: it may be 0.

Hint 3 Take $y = 0$ and $x = -\dfrac{1}{2}$ as two separate cases. You should find four numbers.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can add, subtract, multiply and divide complex numbers in the form $a + bi$. ○ ○ ○

I can divide by multiplying the top and bottom by the conjugate of the bottom. ○ ○ ○

I can solve an equation in z and z^* by equating real and imaginary parts. ○ ○ ○

I can turn a condition such as “is real” into an equation and solve it. ○ ○ ○

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