Cubic and quartic equations with complex roots

97 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-complex-polynomials/

Date

A Do Now

Quick questions to start.

A1 From A-Level Maths [3 marks]

Show that $x = 2$ is a root of $x^3 - 3x^2 - 4x + 12 = 0$, and factorise $x^3 - 3x^2 - 4x + 12$ fully.

Answer .....

A2 Last lesson [2 marks]

Solve $z^2 + 4z + 13 = 0$.

Answer .....

A3 From A-Level Maths [2 marks]

Given that $2x^3 - x^2 + 2x - 1 = (2x - 1)q(x)$, find $q(x)$.

Answer .....

A4 Needed today [2 marks]

Write $(1 + i)^3$ in the form $a + bi$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Showing a root and solving a cubic

EXAMPLE 1 · I DO

Show that $2 + i$ is a root of $z^3 - 3z^2 + z + 5 = 0$, and write down another root.

Hint Work out $(2 + i)^2$ and $(2 + i)^3$ first. Then substitute and collect the real parts and the imaginary parts separately.

YOUR TURN · WE DO

B1 In this question you must show detailed reasoning. Show that $1 - 3i$ is a root of $z^3 - 4z^2 + 14z - 20 = 0$, and write down another root. [4 marks]

Answer .....

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EXAMPLE 2 · I DO

Given that $1 + 4i$ is a root of $z^3 + 13z + 34 = 0$, solve the equation.

Hint The conjugate $1 - 4i$ is also a root. Multiply the two factors to get a real quadratic factor, then find the linear factor.

YOUR TURN · WE DO

B2 Given that $-1 + 2i$ is a root of $z^3 + z^2 + 3z - 5 = 0$, solve the equation. [5 marks]

Answer .....

PRACTICE · YOU DO

C1 Show that $2 - i$ is a root of $z^3 - z^2 - 7z + 15 = 0$, and write down another root. [4 marks]

Answer .....

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C2 A cubic equation with real coefficients and leading coefficient 1 has roots 3 and $2 - 5i$. Find the equation in the form $z^3 + bz^2 + cz + d = 0$. [4 marks]

Answer .....

C3 Given that $1 + 3i$ is a root of $z^3 + 6z + 20 = 0$, solve the equation. [5 marks]

Answer .....

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PART 2 Cubics and quartics

EXAMPLE 3 · I DO

Given that $1 + i$ is a root of $2z^3 - 3z^2 + 2z + 2 = 0$, solve the equation.

Hint The real factor will not be $z + \ldots$: the leading coefficient is 2, so it is $2z + \ldots$.

YOUR TURN · WE DO

B3 Given that $-1 + i$ is a root of $3z^3 + 4z^2 + 2z - 4 = 0$, write $3z^3 + 4z^2 + 2z - 4$ as a product of a quadratic factor and a linear factor with integer coefficients, and solve the equation. [5 marks]

Answer .....

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EXAMPLE 4 · I DO

Given that $1 + 2i$ is a root of $z^4 - 2z^3 + 6z^2 - 2z + 5 = 0$, solve the equation.

Hint The conjugate pair gives one quadratic factor. Compare coefficients to find the other quadratic factor, then solve it.

YOUR TURN · WE DO

B4 In this question you must show detailed reasoning. Given that $-2 + i$ is a root of $z^4 + 2z^3 + 2z + 15 = 0$, find all the roots. [6 marks]

Answer .....

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C4

Sam writes: "$4z^3 - 13z^2 + 8z + 15 = (z^2 - 4z + 5)\left(z + \frac{3}{4}\right)$, so the roots are $2 \pm i$ and $-\frac{3}{4}$."

$2 + i$ is a root of $4z^3 - 13z^2 + 8z + 15 = 0$.

(a) Explain what is wrong with Sam's factorisation. [2 marks]

(b) Write $4z^3 - 13z^2 + 8z + 15$ as a product of factors with integer coefficients. [2 marks]

Answer .....

Total for C4: 4 marks

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C5 Given that $2 + 3i$ is a root of $2z^3 - 7z^2 + 22z + 13 = 0$, solve the equation. [5 marks]

Answer .....

C6 In this question you must show detailed reasoning. Show that $1 + i$ is a root of $z^4 - 3z^3 + 5z^2 - 4z + 2 = 0$, and find the other roots. [6 marks]

Answer .....

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PART 3 Finding unknown coefficients

EXAMPLE 5 · I DO

$1 - 2i$ is a root of $z^3 + pz^2 + qz + 15 = 0$, where p and q are real. Find p, q and the real root.

Hint Find the quadratic factor from the conjugate pair. The constant term 15 tells you the linear factor.

YOUR TURN · WE DO

B5 $3 + i$ is a root of $z^3 + pz^2 + qz - 20 = 0$, where p and q are real. Find p, q and the other two roots. [6 marks]

Answer .....

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C7 $3i$ is a root of $z^3 - 2z^2 + kz - 18 = 0$, where k is real. Find k and the other two roots. [4 marks]

Answer .....

C8 A student says a cubic equation with real coefficients can have the roots $1 + i$, $1 - i$ and $2i$.

(a) Explain why the student is wrong. [2 marks]

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(b) The cubic $z^3 + bz^2 + cz - 10 = 0$, where b and c are real, has roots $1 \pm i$. Find b and c. [2 marks]

Answer .....

Total for C8: 4 marks

D Exam-style questions

OCR exam style

D1

> Alex writes: "The coefficients are real, so the roots come in conjugate pairs. So $1 + i$ and $1 - i$ are both roots of $f(z) = 0$."

$f(z) = z^3 + z^2 - 4z + 6$.

(a) Explain why Alex's argument does not show that $1 + i$ is a root. [2 marks]

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(b) Show that $1 - i$ is a root of $f(z) = 0$. [3 marks]

(c) Hence solve $f(z) = 0$. [3 marks]

Answer .....

Total for D1: 8 marks

D2 A cubic polynomial $f(z)$ has real coefficients and leading coefficient 1.
Two of the roots of $f(z) = 0$ are $2 - i$ and the real number k.

(a) Show that $f(z) = z^3 - (k+4)z^2 + (4k+5)z - 5k$. [3 marks]

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(b) Given that $f(1) = 6$, find k. [2 marks]

Answer .....

(c) Given instead that $f(i)$ is purely imaginary, find k and the value of $f(i)$. [4 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find every real number a for which $z = a + ai$ is a root of $z^3 - 4z^2 + 6z - 4 = 0$. Hence solve the equation. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Work out $(a+ai)^2$ and $(a+ai)^3$ in terms of a.

Hint 2 Both the real part and the imaginary part must be zero. Solve the simpler one first.

Hint 3 The imaginary part gives three values of a, including $a=0$. Check each in the real part.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can show that a complex number is a root of a polynomial by showing the real and imaginary parts are both zero.	○ ○ ○
I can use a conjugate pair of roots to find a quadratic factor with real coefficients.	○ ○ ○
I can factorise a real cubic or quartic and find all its roots.	○ ○ ○
I can find unknown coefficients of a real polynomial from one complex root.	○ ○ ○

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