Square roots of complex numbers and quadratic equations

93 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-complex-roots/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Write $(3 - i)^2$ in the form $a + bi$.

Answer .....

A2 From A-Level Maths [2 marks]

Write $x^2 - 6x + 13$ in the form $(x + p)^2 + q$.

Answer .....

A3 Last lesson [2 marks]

Solve $2z - z^* = 3 + 6i$.

Answer .....

A4 Needed today [2 marks]

Find the real values of x for which $x^4 - 5x^2 - 36 = 0$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Quadratic equations with complex roots

EXAMPLE 1 · I DO

Solve $z^2 - 4z + 13 = 0$.

Hint Complete the square. Then use $\sqrt{-9} = 3i$.

EXAMPLE 2 · I DO

Solve $2z^2 - 2z + 5 = 0$. Give the roots in the form $p \pm qi$.

Hint Use the quadratic formula. Divide both the real part and the imaginary part by the bottom.

YOUR TURN · WE DO

B1 In this question you must show detailed reasoning. Solve $z^2 + 6z + 34 = 0$. [4 marks]

Answer .....

YOUR TURN · WE DO

B2 Solve $4z^2 - 12z + 25 = 0$. Give the roots in the form $p \pm qi$. [4 marks]

Answer .....

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C1 Solve $z^2 + 8z + 25 = 0$. [3 marks]

Answer .....

C2 In this question you must show detailed reasoning. $z = 1 - 4i$ is a root of $z^2 + pz + q = 0$, where p and q are real. Find p and q, and the other root. [4 marks]

Answer .....

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C3 Find the set of values of the real constant k for which $z^2 + kz + 9 = 0$ has two roots that are not real. Solve the equation when $k = 2$. [5 marks]

Answer .....

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PART 2 Square roots of a complex number

EXAMPLE 3 · I DO

Find the two square roots of $15 + 8i$.

Hint Let a square root be $a + bi$ with a and b real. Square it and equate the real parts and the imaginary parts.

YOUR TURN · WE DO

B3 Find the two square roots of $-5 + 12i$. [5 marks]

Answer .....

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EXAMPLE 4 · I DO

Find the two square roots of $1 + 2\sqrt{2}\,i$.

Hint The method is the same. The surd stays in ab: here $ab = \sqrt{2}$.

YOUR TURN · WE DO

B4 Find the two square roots of $2 - 4\sqrt{2}\,i$. [5 marks]

Answer .....

PRACTICE · YOU DO

C4 Find the two square roots of $12 + 16i$. [4 marks]

Answer .....

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C5 Sam finds the square roots of $21 - 20i$. He correctly gets $a^2 - b^2 = 21$, $ab = -10$ and $a^2 = 25$. He writes the answer as $\pm 5 \pm 2i$. Explain what is wrong with Sam's answer, and write down the two square roots. [4 marks]

Answer .....

C6 Write $\dfrac{7+i}{1-i}$ in the form $a + bi$. Hence find the two square roots of $\dfrac{7+i}{1-i}$. [5 marks]

Answer .....

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PART 3 Solving quadratics with a complex square root

EXAMPLE 5 · I DO

Solve $z^2 + 2z + 1 - 2i = 0$.

Hint Complete the square. Then find the two square roots of the complex number on the right.

YOUR TURN · WE DO

B5 Solve $z^2 - 6z + 12 - 4i = 0$. [6 marks]

Answer .....

PRACTICE · YOU DO

C7 Solve $(z + 1)^2 = 3 + 4i$. [4 marks]

Answer .....

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C8 Solve $z^2 - 4z + 13 - 40i = 0$. [5 marks]

Answer .....

D Exam-style questions

OCR exam style

D1

> Kim writes: "Let a root be $a + bi$. Then $a^2 - b^2 = -7$ and $ab = -12$, so $a^4 + 7a^2 - 144 = 0$. So $a^2 = 9$ or $a^2 = -16$, which gives $a = \pm 3$ or $a = \pm 4i$. So $-7 - 24i$ has four square roots."

Kim is finding the square roots of $-7 - 24i$.

(a) Explain the mistake in Kim's working. [2 marks]

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(b) Find the two square roots of $-7 - 24i$. [3 marks]

Answer .....

(c) Hence solve $z^4 + 14z^2 + 625 = 0$. [5 marks]

Answer .....

Total for D1: 10 marks

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D2 In this question you must show detailed reasoning. Solve the equation [8 marks]

$$z^2 - (5+i)z + 8 + i = 0.$$

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 t is a non-zero real number. Solve the equation $z^2 + 2z + 2 - t^2 - 2ti = 0$, giving the roots in terms of t. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Complete the square in z, as you would with numbers.

Hint 2 Find the square roots of the right-hand side by writing them as $x + iy$. The equations now contain t.

Hint 3 The quartic in x factorises, and one factor is $x^2 - t^2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve a quadratic equation with real coefficients and complex roots, giving the roots as $p \pm qi$. ○ ○ ○

I can find the two square roots of a complex number by squaring $a + bi$ and equating parts. ○ ○ ○

I can reject the values of a that are not real when I solve the quartic. ○ ○ ○

I can use square roots of complex numbers to solve harder quadratic and quartic equations. ○ ○ ○

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