A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 1: COMPLEX NUMBERS · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/ocr-further-maths-core-pure-1-chapter-1-a-star/

Complex numbers: A* questions

73 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 In this question you must show detailed reasoning. [8 marks]

The equation

$$z^2 - (2 + 3i)z + k(1 + i) = 0,$$

where k is a non-zero real number, has a real root.

Find the value of k and both roots of the equation.

Answer $k = \ldots$, roots $\ldots$

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X2 In this question you must show detailed reasoning. [8 marks]

Find every complex number z that satisfies

$$z^3 + 8z^* = 0.$$

Give your answers in the form $a + bi$, where a and b are real.

Answer $z = \ldots$

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X3 In this question you must show detailed reasoning. [10 marks]

The complex numbers z and w satisfy

$$z + w = 3 + i \quad \text{and} \quad z^2 + w^2 = -8 + 8i.$$

Find z and w, giving each in the form $a + bi$.

Answer $z, w = \ldots$

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X4 In this question $z = a + bi$ is a non-zero complex number, where a and b are real.

(a) Show that the imaginary part of $z + \frac{1}{z}$ is $b\left(1 - \frac{1}{a^2 + b^2}\right)$. [3 marks]

(b) Prove that $z + \frac{1}{z}$ is real if and only if z is real or $|z| = 1$. [4 marks]

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(c) Find the non-real values of z for which $z + \frac{1}{z} = \frac{6}{5}$. [3 marks]

Answer $z = \ldots$

Total for X4: 10 marks

X5 The quadratic equation

$$z^2 - 2kz + 2k + 12 = 0,$$

where k is real, has roots α and α^*.

(a) Find the set of values of k for which the roots are not real. [3 marks]

Answer $\cdots < k < \ldots$

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(b) It is given that α^2 is purely imaginary. Find the possible values of k and, for each one, the roots of the equation. [4 marks]

Answer $k = \ldots$, roots $\ldots$

(c) It is given instead that α^3 is real. Find the possible values of k and, for each one, the value of α^3. [5 marks]

Answer $k = \ldots$, $\alpha^3 = \ldots$

Total for X5: 12 marks

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X6 The polynomial $f(z)$ is given by

$$f(z) = z^4 - 8z^3 + 30z^2 - 56z + 65.$$

It is given that $a + 2i$ is a root of $f(z) = 0$, where a is real.

(a) Show that a satisfies both [4 marks]

$$a^4 - 8a^3 + 6a^2 + 40a - 39 = 0 \quad \text{and} \quad a^3 - 6a^2 + 11a - 6 = 0.$$

(b) Find the possible values of a. You must justify your answer. [4 marks]

Answer $a = \ldots$

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(c) Hence write $f(z)$ as the product of two quadratic factors with real coefficients, and solve $f(z) = 0$. [4 marks]

Answer $f(z) = \ldots$; $z = \ldots$

Total for X6: 12 marks

X7 In this question you must show detailed reasoning.

(a) Show that $(1 + 2i)^3 = -11 - 2i$. [2 marks]

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(b) Hence find the complex numbers p and q such that [3 marks]

$$z^3 + 11 + 2i = (z - 1 - 2i)(z^2 + pz + q).$$

Answer $p = \ldots$, $q = \ldots$

(c) Find the square roots of $9 - 12i$, giving them in the form $x + yi$, where x and y are real and exact. [4 marks]

Answer $\pm(\ldots)$

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(d) Hence solve the equation $z^3 + 11 + 2i = 0$. Give each root in the form $x + yi$, where x and y are real and exact. [4 marks]

Answer $z = \ldots$

Total for X7: 13 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can equate real and imaginary parts to find unknowns, keeping track of every case	○	○	○
I can find the square roots of a complex number and solve quadratics with complex coefficients	○	○	○
I can prove an if and only if result in both directions	○	○	○
I can find every root of a polynomial from one root, and justify each value I find	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Call the real root x. Put $z = x$ into the equation.
	Hint 2	Collect the real terms and the terms with i. Each group must be zero.
	Hint 3	One case gives $k = 0$, which the question does not allow. For the second root, use the sum of the roots.
X2	**Hint 1**	Write $z = a + bi$ and expand z^3. Then equate the real parts and the imaginary parts.
	Hint 2	Each equation has a common factor. This gives cases: $a = 0$, $b = 0$, or neither.
	Hint 3	When neither a nor b is zero you have two equations in a^2 and b^2. Count your solutions at the end.
X3	**Hint 1**	Find zw from $(z + w)^2$ and $z^2 + w^2$.
	Hint 2	Once you know $z + w$ and zw, z and w are the two roots of one quadratic.
	Hint 3	The discriminant is not real. Find its square root by writing $(p + qi)^2$ equal to it.
X4	**Hint 1**	For $1/z$, multiply the top and the bottom by z^*.
	Hint 2	For (b), prove the two directions separately: "if" and "only if".
	Hint 3	In (c), z is not real, so part (b) tells you $\lvert z \rvert$. What is $1/z$ then?
X5	**Hint 1**	Write $\alpha = k + si$. The product of the roots gives you s^2 in terms of k.
	Hint 2	In (b), "purely imaginary" means the real part of α^2 is zero. In (c), "real" means the imaginary part of α^3 is zero.
	Hint 3	Part (c) says "instead": drop the condition from (b). Check each k against your answer to (a).
X6	**Hint 1**	Expand $(a + 2i)^2$, then multiply on to get the cube and the fourth power.
	Hint 2	A value of a must make both the real part and the imaginary part zero.
	Hint 3	f has real coefficients, so each non-real root comes with its conjugate.

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X7

Hint 1 For (b), expand the right-hand side and compare the coefficients of z^2, z and the constant.

Hint 2 For (c), write $(x+yi)^2 = 9-12i$ and equate real and imaginary parts. x must be real.

Hint 3 For (d), the discriminant of the quadratic factor is the number in (c). The roots are not a conjugate pair.

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