A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 2: THE ARGAND DIAGRAM · A* EXAM PRACTICE

Hints online, answers free with an account:
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The Argand diagram: A* questions

75 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The complex number w is given by $w = \dfrac{a + 2i}{1 + i}$, where a is real. In this question arguments are in the interval $-\pi < \arg z \leqslant \pi$.

(a) Given that $\arg w = \dfrac{\pi}{3}$, find the exact value of a. [5 marks]

Answer $a = \ldots$

(b) Show that there is no value of a for which $\arg w = -\dfrac{2\pi}{3}$. [3 marks]

Total for X1: 8 marks

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X2 The roots of the equation $z^2 - 4z + k = 0$, where k is real, are shown on an Argand diagram. The region D is $\{z : |z - 2 - i| \leqslant 3\}$. [9 marks]

Find the set of values of k for which both roots lie in D. Give your answer in set notation.

Answer $\{k : \ldots\}$

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X3 The half-line L is the locus $\arg(z-2)=\dfrac{3\pi}{4}$. The circle C is the locus $|z-4i|=1$. [9 marks]

The point P represents z, where z lies on L, and the point Q represents w, where w lies on C.

Find the least possible value of $|z-w|$, and find the values of z and w for which it occurs.

Answer least value ... ; $z=\ldots$, $w=\ldots$

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X4 The complex number z lies on the circle $|z - 2| = 1$. The complex number w is given by $w = (\sqrt{3} + i)z$.

(a) Show that $|w - 2\sqrt{3} - 2i| = 2$, and describe the locus of w. [3 marks]

(b) Find the least value of $|w|$ and the value of w for which it occurs. [3 marks]

Answer least $|w| = \ldots$, $w = \ldots$

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(c) Find the greatest value of $\arg w$, and find the value of w for which it occurs. Give w in the form $x + yi$, where x and y are exact. [5 marks]

Answer greatest $\arg w = \ldots$, $w = \ldots$

Total for X4: 11 marks

X5 The region R is

$$R = \{z : |z - 3| \leqslant 3\} \cap \{z : |z - 3| \geqslant |z - 3 - 3i|\}.$$

(a) Find a cartesian equation of the line that forms part of the edge of R, and say which side of it R lies on and whether the line belongs to R. [3 marks]

Answer $y = \ldots$

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(b) Find the complex numbers at the two corners of R. [2 marks]

Answer ...

(c) Find the exact area of R. [4 marks]

Answer area $=$ ...

(d) Find the greatest value of $\arg z$ for z in R. [3 marks]

Answer greatest $\arg z =$ ...

Total for X5: 12 marks

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X6 In this question z is a non-zero complex number with $|z| = r$ and $\arg z = \theta$.

(a) Show that $\dfrac{z}{z^*} = \cos 2\theta + i \sin 2\theta$. [3 marks]

(b) Prove that $\dfrac{z}{z^*}$ is real if and only if z lies on the real axis or on the imaginary axis. [3 marks]

(c) The region S is $\left\{z : z \text{ is non-zero and } \operatorname{Re}\left(\dfrac{z}{z^*}\right) \geqslant 0\right\}$. Describe S, giving its boundary lines. [2 marks]

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(d) Find the exact area of the part of S that lies inside the circle $|z - 2| = 2$. [4 marks]

Answer area $= \ldots$

Total for X6: 12 marks

X7 The polynomial $f(z)$ is given by

$$f(z) = z^4 + 11z^2 - 10z + 50.$$

It is given that $1 + 2i$ is a root of $f(z) = 0$.

(a) Find the other three roots of $f(z) = 0$. [5 marks]

Answer $z = \ldots$

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(b) The four roots are plotted on an Argand diagram. [4 marks]

(i) Find the area of the quadrilateral whose vertices are the four roots.

(ii) The region R is $\{z : \sqrt{5} \leqslant |z| \leqslant \sqrt{10}\}$. Show that every root lies on the edge of R, and find the exact area of R.

Answer (i) ... (ii) ...

(c) The complex number w is $\dfrac{5+5i}{2+i}$. Determine whether w is in R. [2 marks]

Answer $w = \ldots$; ...

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(d) Find the least value of $|z - 4 - 4i|$ for z in R, and the value of z for which it occurs. [3 marks]

Answer least value ... , $z = ...$

Total for X7: 14 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find an argument in the correct quadrant and use it to find an unknown	○	○	○
I can find least distances and greatest arguments for points on a locus or in a region	○	○	○
I can describe a region in set notation and find its exact area	○	○	○
I can prove an if and only if result using arguments	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Write w in the form $x + yi$ first, with x and y in terms of a.
	Hint 2	The tangent of the argument is $\frac{y}{x}$, but the tangent alone does not fix the quadrant.
	Hint 3	Decide which quadrant each argument needs, and what that says about the signs of x and y.
X2	**Hint 1**	The roots are real or non-real depending on the discriminant. Treat the two cases separately.
	Hint 2	D is a disc. A root is in D when its distance from $2 + i$ is at most 3.
	Hint 3	The two non-real roots are not the same distance from $2 + i$. Use the one that is further away.
X3	**Hint 1**	Write a general point of L as $(2 - t) + ti$. Say which values of t are allowed.
	Hint 2	First find the point of L closest to the centre of C. Check it is on the half-line.
	Hint 3	The closest point of C is on the radius that points towards that point of L.
X4	**Hint 1**	In (a), write $2\sqrt{3} + 2i$ as $(\sqrt{3} + i) \times 2$ and take out a common factor.
	Hint 2	For (b), join O to the centre of the circle for w.
	Hint 3	For (c), draw the tangent from O that touches the upper side of the circle. The radius meets it at a right angle.
X5	**Hint 1**	$\lvert z - 3\rvert = \lvert z - 3 - 3i\rvert$ is a perpendicular bisector. Which side is nearer $3 + 3i$?
	Hint 2	The area is a segment: a sector of the circle minus a triangle.
	Hint 3	For (d), the point of R with the greatest argument is on its edge. Compare the points along the edge, not just the top of the circle.

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X6

Hint 1 For (a), find the modulus and the argument of z^*, then use the rules for a quotient.

Hint 2 For (b), a number $\cos\phi + i\sin\phi$ is real when $\sin\phi = 0$. Prove both directions.

Hint 3 For (d), find where the lines $y = \pm x$ meet the circle. The region inside the circle splits into a triangle and a half-disc.

X7

Hint 1 f has real coefficients, so the conjugate of $1 + 2i$ is also a root.

Hint 2 In (b), the quadrilateral and the region R are different shapes. Find each area separately.

Hint 3 In (d), the point $4 + 4i$ is outside R. The nearest point of R lies on the line through O and $4 + 4i$.

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