A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 3: MATRICES · A* EXAM PRACTICE

Hints online, answers free with an account:
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Matrices: A* questions

71 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 $\mathbf{A} = \begin{pmatrix} 5 & 1 \\ 5 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$. [8 marks]

The 2×2 matrix $\mathbf{X}$ satisfies $\mathbf{XA} = \mathbf{B} + 2\mathbf{X}$.

Find $\mathbf{X}$. Show that $\mathbf{X}$ is not equal to $(\mathbf{A} - 2\mathbf{I})^{-1}\mathbf{B}$.

Answer $\mathbf{X} = \ldots$

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X2 [8 marks]

$\mathbf{N} = \begin{pmatrix} 3 & x-1 & 1 \\ 1 & x-1 & -1 \\ 2 & 1 & x \end{pmatrix}$, where x is real.

Show that $\mathbf{N}$ is non-singular for every real value of x.

Find the value of x for which $\det(\mathbf{N}^{-1})$ is greatest, and state this greatest value.

Answer $x = \ldots$, greatest value $= \ldots$

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X3 $\mathbf{Z} = \begin{pmatrix} z & 1+i \\ 1-i & z^* \end{pmatrix}$, where z is a complex number and z^* is its complex conjugate.

(a) Show that $\det \mathbf{Z}$ is real and equal to $|z|^2 - 2$. [2 marks]

(b) Describe the set of points z in an Argand diagram for which $\mathbf{Z}$ is singular. Give the set in set notation. [2 marks]

Answer $\{z : \ldots\}$

(c) Find $\mathbf{Z}^{-1}$ when $z = 2 - i$. [3 marks]

Answer $\mathbf{Z}^{-1} = \ldots$

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(d) z lies on the line $\mathrm{Re}(z) = \mathrm{Im}(z)$ and $\det \mathbf{Z} = 6$. Find the two possible values of z. [2 marks]

Answer $z = \ldots$ or $z = \ldots$

Total for X3: 9 marks

X4 $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$. A 2×2 matrix $\mathbf{X} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ commutes with $\mathbf{A}$ when $\mathbf{XA} = \mathbf{AX}$.

(a) Show that $\mathbf{X}$ commutes with $\mathbf{A}$ if and only if $r = 2q$ and $p = q + s$. [4 marks]

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(b) Hence show that every matrix that commutes with $\mathbf{A}$ can be written as $q\mathbf{A} + (s - 2q)\mathbf{I}$. [2 marks]

Sam says: "The only matrices that commute with $\mathbf{A}$ are multiples of $\mathbf{A}$ and multiples of $\mathbf{I}$." Explain why Sam is wrong.

(c) Show that $\mathbf{A}^2 = 5\mathbf{A} - 4\mathbf{I}$. [5 marks]

Hence find the two matrices $\mathbf{X}$, other than $\mathbf{0}$ and $\mathbf{I}$, that commute with $\mathbf{A}$ and satisfy $\mathbf{X}^2 = \mathbf{X}$.

Answer $\mathbf{X} = \ldots$ or $\mathbf{X} = \ldots$

Total for X4: 11 marks

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X5 $\mathbf{M} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 0 \\ 1 & 0 & a \end{pmatrix}$, where a is a constant.

(a) Show that $\det \mathbf{M} = a^3 - 2a$. [2 marks]

(b) In this question you must show detailed reasoning. [2 marks]

Find the exact values of a for which $\mathbf{M}$ is singular.

Answer $a = \ldots$

(c) a is an integer. Every entry of $\mathbf{M}^{-1}$ is also an integer. Determine the possible values of a. [4 marks]

Answer $a = \ldots$

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(d) Use $\mathbf{M}^{-1}$, with $a = 1$, to solve [3 marks]

$$x + y + z = 2, \quad x + y = 1, \quad x + z = 0.$$

Answer $x = \ldots,\ y = \ldots,\ z = \ldots$

Total for X5: 11 marks

X6 $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{pmatrix}$, where k is a constant.

(a) Show that $\det \mathbf{A} = (k-1)^2$. State the value of k for which the equations $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$ do not have a unique solution. [3 marks]

Answer $k = \ldots$

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(b) Find $\mathbf{A}^{-1}$ in terms of k, for k not equal to 1. [4 marks]

Answer $\mathbf{A}^{-1} = \ldots$

(c) Hence solve [3 marks]

$$x + y + z = 3, \quad x + ky + z = 5, \quad x + y + kz = 1,$$

giving y and z in terms of k. Show that x does not depend on k.

Answer $x = \ldots,\ y = \ldots,\ z = \ldots$

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(d) The solution also satisfies $x^2 + y^2 + z^2 = 17$. Find the possible values of k. [2 marks]

Answer $k = \ldots$

Total for X6: 12 marks

X7 $\mathbf{N}$ is a 3×3 matrix with $\mathbf{N}^3 = \mathbf{0}$, and $\mathbf{I}$ is the 3×3 identity matrix.

(a) Show that $(\mathbf{I} - \mathbf{N})(\mathbf{I} + \mathbf{N} + \mathbf{N}^2) = \mathbf{I}$. [2 marks]

(b) Explain why $\mathbf{I} - \mathbf{N}$ is non-singular. [2 marks]

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(c) $\mathbf{U} = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$. Find the matrix $\mathbf{N}$ with $\mathbf{U} = \mathbf{I} - \mathbf{N}$, and show that $\mathbf{N}^3 = \mathbf{0}$. [3 marks]

(d) Hence find $\mathbf{U}^{-1}$. [3 marks]

Answer $\mathbf{U}^{-1} = \ldots$

(e) Solve [2 marks]

$$x - 2y + 3z = 3, \quad y - 4z = -2, \quad z = 1.$$

Answer $x = \ldots,\ y = \ldots,\ z = \ldots$

Total for X7: 12 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can solve a matrix equation, keeping the order of multiplication right.	○	○	○
I can work with a determinant in terms of a parameter and use det(AB) = det A det B.	○	○	○
I can find a 3×3 inverse with a parameter and use it to solve three equations.	○	○	○
I can show or disprove a claim about matrices with reasons.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Move every term with $\mathbf{X}$ to one side. $2\mathbf{X}$ is the same as $\mathbf{X}(2\mathbf{I})$.
	Hint 2	Factorise as $\mathbf{X}(\mathbf{A} - 2\mathbf{I})$. The $\mathbf{X}$ is on the left, so the inverse must go on the right of $\mathbf{B}$.
	Hint 3	Work out $\mathbf{B}(\mathbf{A} - 2\mathbf{I})^{-1}$, then compare it with $(\mathbf{A} - 2\mathbf{I})^{-1}\mathbf{B}$.
X2	**Hint 1**	Expand along the first row. Put brackets round every product before you simplify.
	Hint 2	You should get a quadratic in x. Complete the square to show it can never be 0.
	Hint 3	$\det(\mathbf{N}^{-1})$ is the reciprocal of $\det \mathbf{N}$. Since $\det \mathbf{N}$ is always positive, a smaller $\det \mathbf{N}$ gives a larger reciprocal.
X3	**Hint 1**	For a 2×2 matrix, the determinant is $ad - bc$. Use $zz^* = \lvert z\rvert^2$.
	Hint 2	A matrix is singular when its determinant is 0. For the inverse, the leading diagonal of $\mathbf{Z}$ holds z and z^*.
	Hint 3	On the line $\mathrm{Re}(z) = \mathrm{Im}(z)$, write $z = t + ti$, so $\lvert z\rvert^2 = 2t^2$.
X4	**Hint 1**	Multiply both ways round, then set each of the four entries of $\mathbf{XA}$ equal to the matching entry of $\mathbf{AX}$.
	Hint 2	For (b), substitute $p = q + s$ and $r = 2q$ into $\mathbf{X}$ and split it into a multiple of $\mathbf{A}$ plus a multiple of $\mathbf{I}$.
	Hint 3	For (c), write $\mathbf{X} = \alpha\mathbf{A} + \beta\mathbf{I}$, replace $\mathbf{A}^2$, then compare the coefficients of $\mathbf{A}$ and of $\mathbf{I}$.
X5	**Hint 1**	For (a), expand along the first row, keeping the signs $+, -, +$.
	Hint 2	For (c), the determinant of an integer matrix is an integer, and $\det \mathbf{M} \times \det(\mathbf{M}^{-1}) = 1$.
	Hint 3	So $\det \mathbf{M} = 1$ or -1. Each gives a cubic in a: test small integers to find a factor, then factorise fully.

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X6

Hint 1 There is no unique solution exactly when $\mathbf{A}$ is singular.

Hint 2 Find all nine cofactors, remembering the pattern of signs. Then transpose and divide by the determinant.

Hint 3 For (c), multiply $\mathbf{A}^{-1}$ by the column of right-hand sides. One entry has a factor $k - 1$ on top that cancels.

X7

Hint 1 Expand the brackets term by term. Keep $\mathbf{N}$ on the side it starts on, then use $\mathbf{N}^3 = \mathbf{0}$.

Hint 2 For (b), take the determinant of both sides of (a) and use $\det(\mathbf{PQ}) = \det \mathbf{P} \det \mathbf{Q}$.

Hint 3 For (d), part (a) tells you that the inverse of $\mathbf{I} - \mathbf{N}$ is $\mathbf{I} + \mathbf{N} + \mathbf{N}^2$.

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