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Matrix transformations: A* questions

77 marks

Date

X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 In this question you must show detailed reasoning. [9 marks]

The transformation T is represented by $\mathbf{M} = \begin{pmatrix} 2 & k \\ 1 & 1 \end{pmatrix}$, where k is a constant.

Find the set of values of k for which T has exactly two invariant lines through the origin.

Answer $\{k : \ldots\}$

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X2 $\mathbf{A} = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ represents a reflection in the line $y = \sqrt{3}x$. $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$. [10 marks]

Find the matrix of the transformation represented by $\mathbf{A}$ followed by $\mathbf{B}$, and describe this transformation fully. Find the least positive integer n for which the nth power of this matrix is $\mathbf{I}$.

Hence describe fully the single transformation represented by $(\mathbf{AB})^{10}$.

Answer $n = \ldots$; $(\mathbf{AB})^{10}$ is $\ldots$

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X3 The circle C has equation $x^2 + y^2 - 6x + 4y + 4 = 0$.

The transformation T is a stretch parallel to the x-axis with scale factor 2, followed by a shear with the x-axis invariant that maps the point $(0, 1)$ to $(3, 1)$.

(a) Find the matrix that represents T. [3 marks]

Answer ...

(b) Find the exact area of the image of the region inside C. [2 marks]

Answer Area $=$ ...

(c) Find the equation of the image of C in the form $x^2 + axy + by^2 + cx + dy + e = 0$. [4 marks]

Answer $\cdots = 0$

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(d) The image of the region inside C is then transformed by a matrix $\mathbf{Q}$, with $\det \mathbf{Q} = k$. The final region has area 54π, and its orientation is the reverse of the region inside C. Find k. [2 marks]

Answer $k = \ldots$

Total for X3: 11 marks

X4 The transformation T is a rotation of 60° anticlockwise about the x-axis, followed by a reflection in the plane $z = 0$.

(a) Find the matrix $\mathbf{T}$ that represents T, giving exact entries. [3 marks]

Answer $\mathbf{T} = \ldots$

(b) Show that $\mathbf{T}^2 = \mathbf{I}$. [3 marks]

Sam says: "T includes a rotation of 60°, so the least positive n with $\mathbf{T}^n = \mathbf{I}$ is $n = 6$." Explain why Sam is wrong.

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(c) A cube of side 3 cm is transformed by T. State the volume of its image and the effect of T on orientation. [2 marks]

Answer Volume $= \ldots$ cm³

(d) The point P is transformed by T a total of 2027 times, and its final image is $(2, \sqrt{3}, 1)$. Find the coordinates of P. [3 marks]

Answer $P = (\ldots)$

Total for X4: 11 marks

X5 The transformation T is a reflection in a line L through the origin. T maps the point $P(4, 2)$ to the point $P'(-2, 4)$.

(a) Find the equation of L. [3 marks]

Answer ...

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(b) Find the matrix $\mathbf{M}$ that represents T. [3 marks]

Answer $\mathbf{M} = \ldots$

(c) S is a rotation of 90° anticlockwise about the origin. Find the matrix of T followed by S. Show that this matrix represents a reflection, and find the equation of its mirror line. [4 marks]

Answer Mirror line: ...

(d) Find all the invariant lines of the transformation in part (c). [2 marks]

Answer ...

Total for X5: 12 marks

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X6 S is a shear with the y-axis invariant that maps $(1, 0)$ to $(1, k)$, where k is a constant. R is a rotation of 90° anticlockwise about the origin. The transformation T is S followed by R.

(a) Show that T is represented by $\begin{pmatrix} -k & -1 \\ 1 & 0 \end{pmatrix}$. [2 marks]

(b) Find the value of k for which T has a line of invariant points, and find the equation of that line. [4 marks]

Answer $k = \ldots$, line: $\ldots$

(c) For this value of k, show that every line $y = x + c$ is invariant under T. Explain why only one of these lines is a line of invariant points. [3 marks]

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(d) Show that T has no other invariant lines. [3 marks]

Total for X6: 12 marks

X7

The transformation T is represented by $\mathbf{M} = \begin{pmatrix} 1 & a & 0 \\ a & 1 & a \\ 0 & a & 1 \end{pmatrix}$, where a is a constant.

(a) Show that $\det \mathbf{M} = 1 - 2a^2$. [2 marks]

(b) Find the set of values of a for which T reverses orientation. Give exact values. [3 marks]

Answer $\{a : \ldots\} \cup \{a : \ldots\}$

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(c) The value of a is 1.5, correct to 1 decimal place. A solid of volume 10 cm³ is transformed by T. [4 marks]

Ali says: "The volume of the image is certainly less than 36 cm³." Determine whether Ali is right.

(d) The transformation U is a rotation about the z-axis followed by an enlargement, centre the origin, with scale factor 2. A solid of volume 3 cm³ is transformed by T followed by U. Its image has volume 96 cm³ and its orientation is reversed. Find the possible values of a. [3 marks]

Answer $a = \ldots$

Total for X7: 12 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find the matrix of a combined transformation and describe it fully.	○	○	○
I can find invariant points and invariant lines, including lines not through the origin.	○	○	○
I can use a determinant for area or volume and orientation, including with bounds.	○	○	○
I can use 3D rotations about the axes and reflections in the planes x = 0, y = 0, z = 0.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Take a point $(1, m)$ on $y = mx$. Its image must also lie on $y = mx$.

Hint 2 You get a quadratic in m. Two distinct real roots give two lines. Take care when $k = 0$, as the equation is not then a quadratic.

Hint 3 The line $x = 0$ cannot be written as $y = mx$. Find the image of $(0, 1)$ to see when $x = 0$ is invariant.

X2

Hint 1 "$\mathbf{A}$ followed by $\mathbf{B}$" means $\mathbf{A}$ acts first, so the matrix is $\mathbf{BA}$.

Hint 2 Compare your matrix with $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Which sign is in which corner tells you the sense of the rotation.

Hint 3 A reflection is its own inverse, so $\mathbf{AB} = (\mathbf{BA})^{-1}$.

X3

Hint 1 Write the matrix of each transformation. The one that acts first goes on the right.

Hint 2 Complete the square to find the radius of C. The area is multiplied by $|\det T|$.

Hint 3 For (c), solve $X = 2x + 3y$, $Y = y$ for x and y, then substitute into the equation of C.

X4

Hint 1 Write each matrix. The rotation acts first, so its matrix goes on the right.

Hint 2 Square $\mathbf{T}$. Think about what a transformation equal to its own inverse does when it is repeated.

Hint 3 Use $\mathbf{T}^2 = \mathbf{I}$ to simplify $\mathbf{T}^{2027}$, then undo one application of T.

X5

Hint 1 P and P' are the same distance from the mirror, on opposite sides. Which point of PP' must be on the mirror?

Hint 2 A point on the mirror is its own image. Use that point and P to set up a matrix equation for $\mathbf{M}$.

Hint 3 For (d), a reflection also maps every line perpendicular to its mirror onto itself.

X6

Hint 1 For a shear with the y-axis invariant, $(0, 1)$ stays where it is. The second transformation's matrix goes on the left.

Hint 2 For a line of invariant points, $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ must give one equation, not just the origin.

Hint 3 For (d), map a general point $(t, mt + c)$ and say the image is on the same line for every t. Do not forget lines $x = d$.

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X7

Hint 1 Orientation is reversed when the determinant is negative, not just when it is non-zero.

Hint 2 For (c), find the bounds for a, then the volume at each bound. Is 36 inside the interval?

Hint 3 For (d), the determinant of a combined transformation is the product of the determinants. An enlargement with scale factor 2 in 3D multiplies volume by 2^3.

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