A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 5: FURTHER VECTORS · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/ocr-further-maths-core-pure-1-chapter-5-a-star/

# Further vectors: A* questions

76 marks

Date


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## X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

**X1** The line $L$ has equation $\dfrac{x+1}{2} = \dfrac{y+2}{-1} = \dfrac{z}{2}$. The point $P$ has coordinates $(5, 4, 6)$. Find the coordinates of the point on $L$ that is closest to $P$, and the shortest distance from $P$ to $L$. Find also the coordinates of the two points on $L$ that are at a distance of $9$ from $P$. [9 marks]

Answer Closest point $(\ldots)$, distance $= \ldots$; points $(\ldots)$ and $(\ldots)$



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**X2** The lines $L_1$ and $L_2$ have equations $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.

**(a)** Show that $L_1$ and $L_2$ meet, and find the coordinates of the point $I$ where they meet. [4 marks]

Answer $I = (\ldots)$

**(b)** Find the acute angle between $L_1$ and $L_2$. [3 marks]

Answer $\ldots^\circ$


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**(c)** Find a vector equation of the line through $I$ that is perpendicular to both $L_1$ and $L_2$. [3 marks]

Answer $\mathbf{r} = \ldots$

**Total for X2: 10 marks**


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**X3** The points $A$ and $B$ have coordinates $(1, 0, 3)$ and $(5, 4, -1)$. The line $L$ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -8 \\ -4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$. The point $C$ lies on $L$ and $CA = CB$. Find the coordinates of $C$, the angle $ACB$, and the exact area of triangle $ABC$. [10 marks]

Answer $C = (\ldots)$ , angle $ACB = \ldots^\circ$ , area $= \ldots$


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**X4** The line $L_1$ has equation $x + 4 = \dfrac{y-2}{2} = \dfrac{2z-1}{4}$. The line $L_2$ has equation $\mathbf{r} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} + u \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$. The point $P$ has coordinates $(2, 1, -3)$. The line $M$ passes through $P$, is perpendicular to $L_1$ and meets $L_2$. Find a cartesian equation of $M$, and the acute angle between $M$ and $L_2$. [10 marks]

Answer $M$ : ... , angle $= \ldots^\circ$


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**X5** The top of a vertical mast is at $T(0, 0, 30)$, where distances are in metres and the $z$-axis is vertical. A drone flies in a straight line. At time $t$ seconds its position vector is $\mathbf{r} = (-57\mathbf{i} + 29\mathbf{j} + 20\mathbf{k}) + t(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. A safety rule says the drone must never be within $10\ \text{m}$ of $T$.

**(a)** Find the time at which the drone is closest to $T$, and its position at that time. [4 marks]

Answer $t = \ldots$ s, position $(\ldots)$

**(b)** Show that the drone breaks the safety rule. [2 marks]


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**(c)** Find for how long the drone is within $10\ \mathrm{m}$ of $T$. [3 marks]

Answer ... seconds

**(d)** The drone is modelled as a particle moving in a straight line. Give two reasons why the true closest distance between the drone and the top of the mast could be less than your answer to part (b). [2 marks]

**Total for X5: 11 marks**


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**X6** The lines $L_1$ and $L_2$ have equations $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} + u \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

**(a)** Show that $L_1$ and $L_2$ are skew. [3 marks]

**(b)** Find a vector that is perpendicular to both lines. [2 marks]

Answer ...


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**(c)** The point $P$ on $L_1$ and the point $Q$ on $L_2$ are such that $PQ$ is perpendicular to both lines. Find the coordinates of $P$ and of $Q$. [5 marks]

Answer $P = (\ldots)$ , $Q = (\ldots)$

**(d)** Hence find the length $PQ$. [2 marks]

Answer $PQ = \ldots$

**Total for X6: 12 marks**


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**X7** The points $A$ and $B$ have coordinates $(1, 0, 2)$ and $(5, 4, 0)$. $ABCD$ is a square and $C$ lies on the line $L$ with equation $\mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ -7 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

**(a)** Use the fact that angle $ABC$ is a right angle to find the coordinates of $C$. [3 marks]

Answer $C = (\dots)$

**(b)** Show that $AB = BC$, and find the coordinates of $D$. [2 marks]

Answer $D = (\dots)$


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**(c)** Find a vector of length $1$ that is perpendicular to the plane of the square. [3 marks]

Answer ...

**(d)** The square is the base of a pyramid $ABCDE$. The vertex $E$ lies on the line through the centre of the square that is perpendicular to the square. The volume of the pyramid is $72$. Given that $E$ has a positive $y$-coordinate, find the coordinates of $E$. [6 marks]

Answer $E = (\ldots)$

**Total for X7: 14 marks**


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## F How did it go?

Colour one circle on each line.

| | Not yet | Nearly | Yes |
|---|---|---|---|
| I can find the point on a line closest to a given point, and the points on the line a given distance away. | ○ | ○ | ○ |
| I can decide whether two lines meet or are skew, and find the acute angle between them. | ○ | ○ | ○ |
| I can use the vector product to find a direction perpendicular to two lines, including the common perpendicular of two skew lines. | ○ | ○ | ○ |
| I can solve problems about triangles, squares and paths in three dimensions using scalar and vector products. | ○ | ○ | ○ |


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# Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

**X1**

**Hint 1** Write $L$ in vector form first. Read the direction from the denominators, with their signs.

**Hint 2** Take a general point $Q$ on $L$. The closest point is where $\overrightarrow{PQ}$ is perpendicular to the direction of $L$.

**Hint 3** For the distance $9$, set $|\overrightarrow{PQ}|^2 = 81$. This gives a quadratic in the parameter.

**X2**

**Hint 1** Equate the three components. Two equations give $s$ and $t$; the third is a check.

**Hint 2** The scalar product of the directions is negative, so the angle it gives is obtuse.

**Hint 3** Take the vector product of the two directions. Watch the sign of the middle component.

**X3**

**Hint 1** Write $C$ in terms of $t$. The condition is about lengths, so compare $CA^2$ and $CB^2$.

**Hint 2** The $t^2$ terms cancel. Then find $\overrightarrow{CA}$ and $\overrightarrow{CB}$ and use the scalar product for the angle.

**Hint 3** For an exact area, find $\sin ACB$ from $\cos ACB$ using $\sin^2 + \cos^2 = 1$.

**X4**

**Hint 1** Write $\dfrac{2z-1}{4}$ as $\dfrac{z-c}{k}$ to read the $z$-part of the direction of $L_1$.

**Hint 2** Let $Q$ be the point where $M$ meets $L_2$, in terms of $u$. Then $\overrightarrow{PQ}$ is a direction of $M$.

**Hint 3** Use $\overrightarrow{PQ} \cdot \mathbf{d}_1 = 0$ to find $u$.

**X5**

**Hint 1** Write the vector from $T$ to the drone in terms of $t$.

**Hint 2** The drone is closest when that vector is perpendicular to the direction of travel.

**Hint 3** For part (c), set the squared distance equal to $100$ and solve the quadratic in $t$.


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**X6**

**Hint 1** Skew means not parallel and not meeting. Show both.

**Hint 2** Write $\overrightarrow{PQ}$ using a general point of each line, with two different parameters.

**Hint 3** $\overrightarrow{PQ}$ has zero scalar product with each direction. That gives two equations in $s$ and $u$.

**X7**

**Hint 1** A right angle at $B$ means $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$. Write $C$ in terms of $t$.

**Hint 2** The vector product of two sides of the square is perpendicular to the square. Divide by its length.

**Hint 3** Find the height from the volume. Then go that distance from the centre in both directions along the unit normal, and test the $y$-coordinate.


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