A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 6: ROOTS OF POLYNOMIALS · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/ocr-further-maths-core-pure-1-chapter-6-a-star/

Roots of polynomials: A* questions

72 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The cubic equation $2x^3 - 5x^2 + 4x - 3 = 0$ has roots α, β and γ.

(a) Write down the value of $\alpha + \beta + \gamma$, and hence write $\alpha + \beta$ in terms of γ. [2 marks]

Answer $\alpha + \beta = \ldots$

(b) Hence, using a suitable substitution, find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. [7 marks]

Answer $\cdots = 0$

Total for X1: 9 marks

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X2 The cubic equation $x^3 - 3x^2 + 5x - 2 = 0$ has roots α, β and γ. Without solving the equation, show that it has exactly one real root, and find the exact value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$. [9 marks]

Answer ...

X3 The quartic equation $x^4 - 2x^3 + ax^2 + bx + 30 = 0$, where a and b are real, has a root $1 + 2\mathrm{i}$.

(a) Write down a second root of the equation, giving a reason. [2 marks]

Answer ...

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(b) Hence find a quadratic factor of $x^4 - 2x^3 + ax^2 + bx + 30$ with real coefficients. [2 marks]

Answer ...

(c) Find the values of a and b, and the other two roots. [5 marks]

Answer $a = \ldots$, $b = \ldots$, roots ...

Total for X3: 9 marks

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X4 The cubic equation $x^3 - 6x^2 + kx - 5 = 0$, where k is real, has roots α, β and γ. One root is the reciprocal of another, so that $\alpha\beta = 1$. Find the value of k and solve the equation. [9 marks]

Answer $k = \ldots$; roots $\ldots$

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X5 The cubic equation $3x^3 + 2x^2 - 4x + 5 = 0$ has roots α, β and γ.

(a) Using the substitution $y = \dfrac{x+1}{x}$, find a cubic equation with integer coefficients whose roots are $\dfrac{\alpha+1}{\alpha}$, $\dfrac{\beta+1}{\beta}$ and $\dfrac{\gamma+1}{\gamma}$. [5 marks]

Answer $\cdots = 0$

(b) Hence find the value of $\dfrac{\alpha+1}{\alpha} + \dfrac{\beta+1}{\beta} + \dfrac{\gamma+1}{\gamma}$. [2 marks]

Answer ...

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(c) Hence find the value of $\dfrac{\alpha}{\alpha+1}+\dfrac{\beta}{\beta+1}+\dfrac{\gamma}{\gamma+1}$. [3 marks]

Answer ...

Total for X5: 10 marks

X6 The quartic equation $x^4-8x^3+px^2+qx-15=0$, where p and q are real, has four real roots. The roots form an arithmetic progression, so they can be written $a-3d$, $a-d$, $a+d$ and $a+3d$, where $d>0$.

(a) Show that $a=2$. [2 marks]

(b) Show that $9d^4-40d^2+31=0$. [4 marks]

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(c) Hence find the two possible values of p. [4 marks]

Answer $p = \ldots$ or $p = \ldots$

(d) Given also that q is an integer, find q and the roots of the equation. [3 marks]

Answer $q = \ldots$; roots $\ldots$

Total for X6: 13 marks

X7 The quartic equation $x^4 - 5x^3 + px^2 - 20x + 16 = 0$, where p is real, has roots α, β, γ and δ, with $\alpha\beta = \gamma\delta$.

(a) Show that $\alpha\beta = 4$. [3 marks]

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(b) Given that $\alpha + \beta = 1$, find p and solve the equation. [5 marks]

Answer $p = \ldots$; roots $\ldots$

(c) Find the set of values of p for which all four roots are real. [5 marks]

Answer $p \ldots$

Total for X7: 13 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the relations between the roots and the coefficients of cubic and quartic equations, including sums of squares.	○	○	○
I can find an equation whose roots are related to the roots of a given equation by a substitution, with integer coefficients.	○	○	○
I can use a condition on the roots, such as a reciprocal pair or an arithmetic progression, to find unknown coefficients.	○	○	○
I can decide how many roots are real, using symmetric functions and complex conjugates.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Each new root is the sum of the three roots minus one of them.
	Hint 2	So $y = \frac{5}{2} - x$. Make x the subject and substitute into the cubic.
	Hint 3	Expand carefully, then multiply through so every coefficient is an integer.
X2	**Hint 1**	Find $\alpha^2 + \beta^2 + \gamma^2$. What can you say if the squares of real numbers add to a negative number?
	Hint 2	A cubic with real coefficients has non-real roots in conjugate pairs.
	Hint 3	Expand the product into four symmetric parts. Square $\alpha\beta + \beta\gamma + \gamma\alpha$ to find $\sum \alpha^2\beta^2$.
X3	**Hint 1**	Think about what real coefficients tell you about non-real roots.
	Hint 2	A quadratic with roots p and q is $x^2 - (p+q)x + pq$.
	Hint 3	For the other pair, use the sum of all four roots and the product of all four roots.
X4	**Hint 1**	Which symmetric function contains the product $\alpha\beta$ on its own?
	Hint 2	Use $\alpha\beta\gamma$ to find γ, then the sum of the roots to find $\alpha + \beta$.
	Hint 3	Write $\alpha\beta + \beta\gamma + \gamma\alpha$ as $\alpha\beta + \gamma(\alpha + \beta)$.
X5	**Hint 1**	Make x the subject of $y = \frac{x+1}{x}$ before you substitute.
	Hint 2	Multiply the whole equation by $(y-1)^3$ to clear the fractions.
	Hint 3	In part (c), each term is the reciprocal of a root of your new equation. Put the sum of reciprocals over one denominator.
X6	**Hint 1**	Use the sum of the roots, then the product of the roots.
	Hint 2	Pair $a - 3d$ with $a + 3d$, and $a - d$ with $a + d$. Each pair multiplies to a difference of two squares.
	Hint 3	For p, use $\sum \alpha\beta = \frac{(\sum \alpha)^2 - \sum \alpha^2}{2}$. There are two cases for d^2; keep both.
X7	**Hint 1**	Write $\sum \alpha\beta\gamma$ as $\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta)$.
	Hint 2	In part (b), α and β are the roots of $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
	Hint 3	In part (c), let $s = \alpha + \beta$ and $t = \gamma + \delta$. Find a quadratic for s and t, and the condition on each sum for real roots.

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