A-LEVEL FURTHER MATHS (OCR A) · CORE PURE 1 · CHAPTER 7: PROOF BY INDUCTION · A* EXAM PRACTICE

Hints online, answers free with an account:
mathswariz.uk/worksheets/ocr-further-maths-core-pure-1-chapter-7-a-star/

Proof by induction: A* questions

71 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Prove by induction that $f(n) = 2^{3n+1} + 3 \times 5^{2n+1}$ is divisible by 17 for every non-negative integer n. Deduce that $f(n)$ is never divisible by 34. [8 marks]

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X2

(a) Prove by induction that $3^n \geq 2n^2 + 1$ for every non-negative integer n. [6 marks]

(b) Find every non-negative integer n for which $3^n = 2n^2 + 1$. Justify your answer. [3 marks]

Answer $n = \ldots$

Total for X2: 9 marks

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X3 Prove by induction that $(2n)! \geq 2^n(n!)^2$ for all positive integers n. [9 marks]

X4

(a) Show that the statement is false when $n = 1$ and true when $n = 2$. [2 marks]

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(b) Prove by induction that $4^n > 3^n + 2^n$ for all integers $n \geq 2$. [6 marks]

(c) Sam says: "The step in part (b) also works when $k = 1$. So the statement is true for all positive integers." Explain why Sam is wrong. [2 marks]

Total for X4: 10 marks

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X5 Let $h(n) = 5^{n+1} - 4n - 5$.

(a) Show that $h(k+1) - 5h(k) = 16(k+1)$. [3 marks]

(b) Hence prove by induction that $h(n)$ is divisible by 16 for every non-negative integer n. [5 marks]

(c) Hence find the remainder when 5^{31} is divided by 16. [2 marks]

Answer ...

Total for X5: 10 marks

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X6 The matrix $\mathbf{M} = \begin{pmatrix} 5 & -4 \\ 1 & 1 \end{pmatrix}$.

(a) Prove by induction that $\mathbf{M}^n = 3^{n-1}\begin{pmatrix} 3+2n & -4n \\ n & 3-2n \end{pmatrix}$ for all positive integers n. [8 marks]

(b) Hence show that $\det(\mathbf{M}^n) = 9^n$. [2 marks]

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(c) Find the positive integer n for which $\mathbf{M}^n$ maps the point $(1, 1)$ to a point on the x-axis, and give that point. [2 marks]

Answer $n = \ldots$, point $(\ldots)$

Total for X6: 12 marks

X7 The matrix $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$.

(a) Prove by induction that $\mathbf{A}^n = \begin{pmatrix} 2 \times 3^n - 2^n & 2^n - 3^n \\ 2 \times 3^n - 2^{n+1} & 2^{n+1} - 3^n \end{pmatrix}$ for all positive integers n. [7 marks]

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(b) Hence show that $\det(\mathbf{A}^n) = 6^n$. [3 marks]

(c) Find the smallest positive integer n for which the top right entry of $\mathbf{A}^n$ is less than -5000. [3 marks]

Answer $n = \ldots$

Total for X7: 13 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove a divisibility result by induction, starting at the right base case and writing f(k + 1) in terms of f(k).	○	○	○
I can prove inequalities with powers and factorials by induction, using the assumption and justifying each comparison.	○	○	○
I can prove a formula for the nth power of a matrix by induction and use it.	○	○	○
I can write a full conclusion and explain why a proof by induction needs a true base case.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	The result starts at $n = 0$, not $n = 1$.
	Hint 2	Write 2^{3k+4} as $8 \times 2^{3k+1}$ and 5^{2k+3} as $25 \times 5^{2k+1}$.
	Hint 3	Split 25 as $8 + 17$, so that $8f(k)$ appears. For the last part, is $f(n)$ odd or even?
X2	**Hint 1**	Start from $3^{k+1} = 3 \times 3^k$ and use the assumption.
	Hint 2	Subtract $2(k+1)^2 + 1$ from $6k^2 + 3$ and factorise what is left.
	Hint 3	For part (b), the difference $4k(k-1)$ is zero only for two values of k.
X3	**Hint 1**	Begin from the $n = k$ case, not from the result you want for $n = k + 1$.
	Hint 2	$(2k+2)! = (2k+2)(2k+1)(2k)!$. Use brackets.
	Hint 3	Write $2^{k+1}\left((k+1)!\right)^2$ as $2 \times 2^k(k+1)^2(k!)^2$, then compare $(2k+2)(2k+1)$ with $2(k+1)^2$.
X4	**Hint 1**	Substitute $n = 1$ and $n = 2$ directly.
	Hint 2	Start from $4^{k+1} = 4 \times 4^k$ and use the assumption.
	Hint 3	4×3^k is bigger than 3×3^k, and 4×2^k is bigger than 2×2^k. For part (c), what does the step on its own prove?
X5	**Hint 1**	Write out $h(k+1)$ and $5h(k)$ in full and subtract.
	Hint 2	In part (b), rearrange part (a) to give $h(k+1)$ in terms of $h(k)$.
	Hint 3	In part (c), choose n so that $5^{n+1} = 5^{31}$.
X6	**Hint 1**	Show the $n = 1$ case by putting $n = 1$ into the formula.
	Hint 2	Multiply $\mathbf{M}^k$ by $\mathbf{M}$, then take out a factor of 3 from every entry.
	Hint 3	$3 \times 3^{k-1} = 3^k$. Then write each entry in terms of $k + 1$.
X7	**Hint 1**	Check $n = 1$ by putting it into every entry.
	Hint 2	Let $a = 3^k$ and $b = 2^k$ to keep the multiplication tidy.
	Hint 3	$2 \times 2^k = 2^{k+1}$, not 4^k. Write each entry of $\mathbf{A}^{k+1}$ in the same form as the formula.

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