Determinants and inverses of 2 by 2 matrices

87 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-det-inverse-2×2/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -2 & 1 \end{pmatrix}$.

Answer .....

A2 From A-Level Maths [2 marks]

Solve $3a^2 - 2a - 8 = 0$.

Answer .....

A3 From A-Level Maths [3 marks]

Write $x^2 - 6x + 11$ in the form $(x - p)^2 + q$.

State its least value.

Answer .....

A4 Needed today [2 marks]

Show that $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} = \mathbf{I}$.

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 The determinant and singular matrices

EXAMPLE 1 · I DO

Find the determinant of $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & -5 \end{pmatrix}$.

Hint For $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant is $ad - bc$. Keep the sign: a determinant can be negative.

YOUR TURN · WE DO

B1 Find $\det \mathbf{P}$ and $\det \mathbf{Q}$, where $\mathbf{P} = \begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 6 & -4 \\ 3 & -2 \end{pmatrix}$. Which of them is singular? [3 marks]

Answer .....

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EXAMPLE 2 · I DO

$\mathbf{M} = \begin{pmatrix} a & 3 \\ 2 & a+5 \end{pmatrix}$. Find the values of a for which $\mathbf{M}$ is singular.

Hint Singular means the determinant is 0. Write the determinant, set it equal to 0 and solve.

YOUR TURN · WE DO

B2 $\mathbf{N} = \begin{pmatrix} k-1 & 4 \\ 3 & k+3 \end{pmatrix}$. Find the values of k for which $\mathbf{N}$ is singular. [4 marks]

Answer .....

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C1 $\mathbf{A} = \begin{pmatrix} 2 & a \\ 3 & 4 \end{pmatrix}$. Sam writes: "$\mathbf{A}$ is singular when $\det \mathbf{A} = 1$, so $8 - 3a = 1$ and $a = \frac{7}{3}$." Explain Sam's mistake and find the correct value of a. [4 marks]

Answer .....

C2 $\mathbf{T} = \begin{pmatrix} 2x & x - 1 \\ x + 1 & x + 3 \end{pmatrix}$, where x is real.

(a) Show that $\det \mathbf{T} = x^2 + 6x + 1$. [2 marks]

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(b) Hence find the least value of $\det \mathbf{T}$, and the value of x where it occurs. [3 marks]

Answer .....

(c) Find the exact values of x for which $\mathbf{T}$ is singular. [2 marks]

Answer .....

Total for C2: 7 marks

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PART 2 The inverse of a 2 by 2 matrix

EXAMPLE 3 · I DO

Find the inverse of $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}$.

Hint Swap the leading diagonal, change the signs of the other two entries, and divide by the determinant.

YOUR TURN · WE DO

B3 Find the inverse of $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}$. [4 marks]

Answer .....

PRACTICE · YOU DO

C3 $\mathbf{A} = \begin{pmatrix} a & 4 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{A}^{-1} = \begin{pmatrix} 1 & -2 \\ -\frac{1}{2} & b \end{pmatrix}$. Find a and b. [4 marks]

Answer .....

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C4 Show that $\begin{pmatrix} 1+i & 2 \\ 1 & 1-i \end{pmatrix}$ is singular. Find the inverse of $\mathbf{W} = \begin{pmatrix} 1+i & i \\ 1 & 1-i \end{pmatrix}$, giving each entry in the form $\frac{p+qi}{5}$. [5 marks]

Answer .....

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PART 3 Rules for determinants and matrix equations

EXAMPLE 4 · I DO

$\mathbf{P}$ is a 2×2 matrix with $\det \mathbf{P} = 5$. Find $\det(\mathbf{P}^3)$, $\det(3\mathbf{P})$ and $\det(\mathbf{P}^{-1})$.

Hint $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$. Multiplying a 2×2 matrix by 3 multiplies both rows by 3, so the determinant is multiplied by 3^2. $\mathbf{PP}^{-1} = \mathbf{I}$ and $\det \mathbf{I} = 1$.

YOUR TURN · WE DO

B4 $\mathbf{N}$ is a 2×2 matrix with $\det \mathbf{N} = -3$. Find $\det(\mathbf{N}^2)$, $\det(2\mathbf{N})$ and $\det(\mathbf{N}^{-1})$. [4 marks]

Answer .....

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EXAMPLE 5 · I DO

$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$. Find the matrix $\mathbf{X}$ for which $\mathbf{AX} = \begin{pmatrix} 4 & 1 \\ 9 & 2 \end{pmatrix}$.

Hint $\mathbf{A}$ is on the LEFT of $\mathbf{X}$, so multiply both sides by $\mathbf{A}^{-1}$ on the left: $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$.

YOUR TURN · WE DO

B5 $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$. Find the matrix $\mathbf{Y}$ for which $\mathbf{YA} = \begin{pmatrix} 1 & 0 \\ 5 & 2 \end{pmatrix}$. [5 marks]

Answer .....

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C5 $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 5 & 0 \end{pmatrix}$. Find $\det \mathbf{A}$, $\det \mathbf{B}$ and $\mathbf{AB}$, and verify that $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$. [5 marks]

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C6 In this question you must show detailed reasoning. $\mathbf{A} = \begin{pmatrix} k & 3 \\ 2 & k+1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & k \\ -1 & 2 \end{pmatrix}$. Find every value of k for which $\mathbf{AB}$ is singular. [7 marks]

Answer .....

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D1 When $a = 3$, Alex writes: "$\det \mathbf{N} = 4$, so $\det(\mathbf{N}^2) = 2 \times 4 = 8$."

$\mathbf{N} = \begin{pmatrix} a & 2 \\ 1 & a-1 \end{pmatrix}$, where a is a constant.

(a) Find $\det \mathbf{N}$ in terms of a, and the values of a for which $\mathbf{N}$ is singular. [3 marks]

Answer .....

(b) Explain Alex's mistake and give the correct value of $\det(\mathbf{N}^2)$. [2 marks]

Answer .....

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(c) Hence, with $a = 3$, find $\mathbf{N}^{-1}$ and $\det(\mathbf{N}^{-1})$. [3 marks]

Answer .....

Total for D1: 8 marks

D2 $\mathbf{A} = \begin{pmatrix} 2 & k \\ 1 & 3 \end{pmatrix}$, where k is a constant.

(a) Find $\mathbf{A}^{-1}$ in terms of k, stating the value of k for which it does not exist. [4 marks]

Answer .....

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(b) Hence, with $k = 4$, find the matrix $\mathbf{X}$ for which $\mathbf{AX} = \begin{pmatrix} 1 & 5 \\ 2 & -1 \end{pmatrix}$. [3 marks]

Answer .....

(c) With $k = 4$, show that $\mathbf{A}^2 - 5\mathbf{A} + 2\mathbf{I} = \mathbf{0}$. Use this to write $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ and $\mathbf{I}$, and check it agrees with part (a). [3 marks]

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $\mathbf{A}$ is a real 2×2 matrix that is not a multiple of $\mathbf{I}$, and $\mathbf{A}^2 = 3\mathbf{A} - 2\mathbf{I}$. Find $\det \mathbf{A}$, write $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ and $\mathbf{I}$, and give an example of such a matrix with no zero entries. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and find $\mathbf{A}^2 - (a + d)\mathbf{A}$. What do you notice?

Hint 2 Compare that with the relation you are given. What does it force $a + d$ to be?

Hint 3 Rearrange the given relation into $\mathbf{A} \times (\text{something}) = \mathbf{I}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the determinant of a 2 by 2 matrix and the values that make it singular.	○ ○ ○
I can find the inverse of a 2 by 2 matrix, including one in terms of a letter.	○ ○ ○
I can solve AX = B and YA = B, multiplying by the inverse on the correct side.	○ ○ ○
I can use det(AB) = det A det B, det(kA) and det(A^{-1}), and show detailed reasoning.	○ ○ ○

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