Determinants and inverses of 3 by 3 matrices

98 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-det-inverse-3×3/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the inverse of $\begin{pmatrix} 4 & -3 \\ 5 & -2 \end{pmatrix}$.

Answer .....

A2 From A-Level Maths [3 marks]

Factorise $a^3 - 2a^2 - 5a + 6$ fully.

Answer .....

A3 Last lesson [2 marks]

The 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$ have $\det \mathbf{A} = 4$ and $\det \mathbf{B} = -3$. Find $\det(\mathbf{AB})$ and $\det(\mathbf{A}^{-1})$.

Answer .....

A4 Needed today [1 mark]

Write down the transpose of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Finding a 3 by 3 determinant

EXAMPLE 1 · I DO

Find the determinant of $\mathbf{A} = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 0 & 2 \\ 1 & 4 & 1 \end{pmatrix}$.

Hint Expand along the top row. Each entry times the determinant of what is left when its row and column are crossed out, with signs $+ \; - \; +$. The middle term is SUBTRACTED.

YOUR TURN · WE DO

B1 Find the determinant of $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \\ 2 & 1 & 5 \end{pmatrix}$. [3 marks]

Answer .....

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EXAMPLE 2 · I DO

Find the determinant of $\begin{pmatrix} 3 & 0 & 2 \\ -1 & 0 & 4 \\ 5 & 2 & 1 \end{pmatrix}$, choosing a good row or column.

Hint Column 2 has two zeros. Expand down it, using the sign pattern for that column: $- + -$.

YOUR TURN · WE DO

B2 Find the determinant of $\begin{pmatrix} 4 & 1 & -2 \\ 0 & 3 & 0 \\ 2 & -1 & 5 \end{pmatrix}$, choosing a good row or column. [3 marks]

Answer .....

PRACTICE · YOU DO

C1 Sam finds the determinant of $\begin{pmatrix} 2 & 1 & 3 \\ 0 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$ as $2(2-8)+1(0-4)+3(0+1)=-13$. Explain Sam's mistake and find the correct determinant. [4 marks]

Answer .....

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C2 [6 marks]

Show that $\det \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} = (b-a)(c-a)(c-b)$.

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PART 2 Singular values and the inverse

EXAMPLE 3 · I DO

Find the values of a for which $\begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$ is singular.

Hint Expand to get a cubic in a. Find one root by trial, factorise, then solve what is left.

YOUR TURN · WE DO

B3 In this question you must show detailed reasoning. Find the exact values of k for which $\begin{pmatrix} k & 1 & 2 \\ 1 & k & 1 \\ 2 & 1 & k \end{pmatrix}$ is singular. [5 marks]

Answer .....

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EXAMPLE 4 · I DO

Find the inverse of $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$.

Hint Find $\det \mathbf{A}$. Find the nine minors, put on the signs $+ \; - \; +$ to get cofactors, TRANSPOSE, then divide by the determinant.

YOUR TURN · WE DO

B4 Find the inverse of $\mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \end{pmatrix}$. [6 marks]

Answer .....

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C3 In this question you must show detailed reasoning. Find the exact values of p for which $\begin{pmatrix} p & 2 & 0 \\ 1 & p & 1 \\ 0 & 3 & p \end{pmatrix}$ is singular. [5 marks]

Answer .....

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C4 [6 marks]

$\mathbf{A} = \begin{pmatrix} 1 & 2 & a \\ 0 & a & 1 \\ 3 & 1 & 2 \end{pmatrix}$, where a is a constant. With $a = 1$, find $\mathbf{A}^{-1}$.

Answer .....

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PART 3 The inverse of a product, volume and orientation

EXAMPLE 5 · I DO

$\mathbf{A}$ and $\mathbf{B}$ are non-singular with $\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ and $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$. Find $(\mathbf{AB})^{-1}$.

Hint The inverse of a product reverses the order: $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$, not $\mathbf{A}^{-1}\mathbf{B}^{-1}$.

YOUR TURN · WE DO

B5 [4 marks]

$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, and $\mathbf{B}$ is a 3×3 matrix with $(\mathbf{AB})^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Find $\mathbf{B}^{-1}$.

Answer .....

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EXAMPLE 6 · I DO

The transformation T has matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 1 & 0 & 2 \end{pmatrix}$. A solid has volume 5 units³. Find the volume of its image under T, and say whether T keeps or reverses orientation. T maps the point P to $(4, -1, -1)$. Find P.

Hint Volume scale factor $= |\det|$. A negative determinant reverses orientation. To find P, use the inverse matrix.

YOUR TURN · WE DO

B6 The transformation S has matrix $\mathbf{S} = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. Find the volume scale factor of S and say what S does to orientation. S maps the point Q to $(4, 5, 3)$. Find Q. [5 marks]

Answer .....

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C5 $\mathbf{A}$ and $\mathbf{B}$ are 3×3 matrices with $\det \mathbf{A} = -2$ and $\det \mathbf{B} = 5$. Find $\det(\mathbf{AB})$, $\det(3\mathbf{A})$ and $\det(\mathbf{A}^{-1}\mathbf{B})$. [4 marks]

Answer .....

C6 The matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & k \end{pmatrix}$ maps the unit cube to a solid of volume 10 units³. Find the possible values of k. [5 marks]

Answer .....

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C7 $\mathbf{M} = \begin{pmatrix} a & 1 & 2 \\ 2 & a & 3 \\ 2 & 1 & a \end{pmatrix}$, where a is a real constant.

(a) Show that $\det \mathbf{M} = a^3 - 9a + 10$. [3 marks]

(b) In this question you must show detailed reasoning. Find the exact values of a for which $\mathbf{M}$ is singular. [4 marks]

Answer .....

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(c) When $a = 0$, M is the matrix of a transformation. Find its volume scale factor and state whether it keeps or reverses orientation. [2 marks]

Answer .....

Total for C7: 9 marks

D Exam-style questions

OCR exam style

D1

Alex writes: "The cofactors of $\mathbf{B}$ are $\begin{pmatrix} 7 & -2 & 1 \\ 1 & 4 & -2 \\ -3 & 3 & 6 \end{pmatrix}$, so $\mathbf{B}^{-1} = \frac{1}{15}\begin{pmatrix} 7 & -2 & 1 \\ 1 & 4 & -2 \\ -3 & 3 & 6 \end{pmatrix}$."

$\mathbf{B} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

(a) Show that $\det \mathbf{B} = 15$. [2 marks]

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(b) Explain Alex's mistake and write down the correct $\mathbf{B}^{-1}$. [3 marks]

Answer .....

(c) Hence find the point that $\mathbf{B}$ maps to $(4, -4, 3)$. [4 marks]

Answer .....

Total for D1: 9 marks

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D2 [8 marks]

The transformation T has matrix $\begin{pmatrix} 2 & 0 & 1 \\ 1 & a & 0 \\ 0 & 1 & 3 \end{pmatrix}$. T maps a solid of volume 4 units³ to a solid of volume 44 units³ and reverses its orientation. T also maps the point Q to $(2, 3, -1)$. Find a and Q.

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [8 marks]

$\mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. Find $\mathbf{M}^{-1}$ and $\mathbf{M}^5$, without working out any 3×3 determinant or cofactor.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Work out $\mathbf{M}^2$. Can you write it using only $\mathbf{M}$ and $\mathbf{I}$?

Hint 2 A relation between $\mathbf{M}^2$, $\mathbf{M}$ and $\mathbf{I}$ can be rearranged into $\mathbf{M} \times (\text{something}) = \mathbf{I}$.

Hint 3 Try writing $\mathbf{M}$ as $\mathbf{I}$ plus the matrix of all 1s. What happens when you square the matrix of all 1s?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find a 3 by 3 determinant, choosing a row or column with zeros.	○ ○ ○
I can find the inverse of a 3 by 3 matrix using cofactors, and use $(AB)^{-1} = B^{-1}A^{-1}$.	○ ○ ○
I can find the values of a letter that make a 3 by 3 matrix singular, showing detailed reasoning.	○ ○ ○
I can use the determinant for a volume scale factor and orientation, and undo a transformation.	○ ○ ○

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