Intersecting, parallel and skew lines

90 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-line-intersections/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the angle between $\begin{pmatrix}1\\0\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\1\end{pmatrix}$.

Answer .....

A2 From A-Level Maths [2 marks]

Solve $2a + b = 7$ and $a - b = -1$.

Answer .....

A3 Needed today [2 marks]

Is $\begin{pmatrix}-3\\6\\-9\end{pmatrix}$ parallel to $\begin{pmatrix}2\\-4\\6\end{pmatrix}$? Give a reason.

Answer .....

A4 Last lesson [1 mark]

Write down the point on $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ where $\lambda = -2$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Lines that meet

EXAMPLE 1 · I DO

In 2-D, find where the lines $\mathbf{r} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}3\\10\end{pmatrix} + \mu\begin{pmatrix}1\\-3\end{pmatrix}$ meet.

Hint Use different letters for the two parameters. Set the x parts equal and the y parts equal, then solve.

YOUR TURN · WE DO

B1 In 2-D, find where the lines $\mathbf{r} = \begin{pmatrix}-2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}2\\-3\end{pmatrix} + \mu\begin{pmatrix}1\\2\end{pmatrix}$ meet. [3 marks]

Answer .....

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EXAMPLE 2 · I DO

Show that the lines $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$ meet, and find the point where they meet.

Hint Solve the x and y equations for λ and μ. Then check that these values also make the z parts equal.

YOUR TURN · WE DO

B2 Show that the lines $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ meet, and find the point where they meet. [5 marks]

Answer .....

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C1 [5 marks]

The lines L_1 and L_2 have equations $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$. Show that L_1 and L_2 meet, and find the coordinates of the point P where they meet.

Answer .....

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C2 Find the coordinates of the point where the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$ and $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z+1}{2}$ meet. [5 marks]

Answer .....

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PART 2 Parallel, the same line, or skew

EXAMPLE 3 · I DO

The lines $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix}$ are given. Are they parallel, or the same line?

Hint Compare the directions: is one a multiple of the other? Then test whether a point of L_2 is on L_1.

YOUR TURN · WE DO

B3 The line L_1 is $\frac{x-1}{3} = \frac{y+2}{-1} = \frac{z-4}{2}$ and L_2 is $\mathbf{r} = \begin{pmatrix} 7 \\ -4 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 2 \\ -4 \end{pmatrix}$. Show that L_1 and L_2 are the same line. [4 marks]

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EXAMPLE 4 · I DO

Show that the lines $L_1 : \mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda \begin{pmatrix}1\\0\\2\end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix}0\\1\\1\end{pmatrix} + \mu \begin{pmatrix}1\\1\\1\end{pmatrix}$ are skew.

Hint Skew means two things: the lines are not parallel, and they have no common point. Show both.

YOUR TURN · WE DO

B4 Show that the lines [5 marks]

$$\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

and $\mathbf{r} = \begin{pmatrix}0\\4\\2\end{pmatrix} + \mu \begin{pmatrix}1\\1\\0\end{pmatrix}$ are skew.

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C3 [4 marks]

Determine whether the line $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ meets the x-axis.

C4 [5 marks]

L_1 is $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and L_2 is $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. Sam writes: "The x and y equations give $\lambda = -2$ and $\mu = -3$, so the lines meet at $(-1, -3, -2)$." Explain Sam's mistake, and decide how the lines are related.

Answer .....

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PART 3 The angle between two lines

EXAMPLE 5 · I DO

The lines $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ k \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ meet. Find k and the point where they meet.

Hint Use the two equations that do not contain k to find λ and μ. The third equation then gives k.

YOUR TURN · WE DO

B5 The lines $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ k \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ meet. Find k and the point where they meet. [5 marks]

Answer .....

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EXAMPLE 6 · I DO

Show that $L_1 : \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$ meet. Find the point where they meet and the acute angle between them.

Hint First find the point as in example 2. For the angle use the two directions only.

YOUR TURN · WE DO

B6 Show that $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ meet. Find the point where they meet and the acute angle between them. [6 marks]

Answer .....

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C5 [4 marks]

L_1 is $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda \begin{pmatrix}2\\a\\-4\end{pmatrix}$ and L_2 is $\mathbf{r} = \begin{pmatrix}3\\1\\0\end{pmatrix} + \mu \begin{pmatrix}-1\\3\\b\end{pmatrix}$. The lines are parallel. Find a and b, and show that the lines are not the same line.

Answer .....

C6 [5 marks]

The lines $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda \begin{pmatrix}2\\1\\-1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}k\\0\\3\end{pmatrix} + \mu \begin{pmatrix}1\\-1\\2\end{pmatrix}$ meet. Find k and the acute angle between the lines.

Answer .....

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D1

> Alex writes: "The directions of L_1 and L_2 are not parallel, so the lines are skew."

The lines L_1 and L_2 have equations $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Explain why Alex's reasoning is not enough. [2 marks]

Answer .....

(b) Show that in fact L_1 and L_2 meet, and find the point where they meet. [5 marks]

Answer .....

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(c) [2 marks]

The line L_3 has equation $\mathbf{r} = \mu \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$. State, with reasons, how L_3 is related to L_1.

Answer .....

Total for D1: 9 marks

D2

The lines L_1 and L_2 have equations $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ k \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, where k is a constant.

(a) The lines meet. Find k. [4 marks]

Answer .....

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(b) Find the point where they meet. [2 marks]

Answer .....

(c) Explain why the lines are skew for every other value of k. [2 marks]

Answer .....

(d) In this question you must show detailed reasoning. Find the exact acute angle between the lines. [2 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [8 marks]

L_1 is the line $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix}$ and L_2 is the line $\mathbf{r} = \begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, where k is a constant. Find every value of k for which L_1 and L_2 meet, and the point where they meet for each one.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Keep k as a letter. Use the x and y equations to write λ and μ in terms of k.

Hint 2 Put these into the z equation. You get an equation in k alone.

Hint 3 Do not divide by $k - 2$: it could be 0. Factorise instead, and find a point for each value.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can show two lines meet by solving two equations and checking the third, and give the point. ○ ○ ○

I can show two lines are skew: not parallel and with no common point. ○ ○ ○

I can tell parallel lines from the same line by testing a point. ○ ○ ○

I can find an unknown so that two lines meet, and the angle between them. ○ ○ ○

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