Loci in the Argand diagram

98 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-loci/

Date

A Do Now

Quick questions to start.

A1 From A-Level Maths [2 marks]

Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$.

Answer .....

A2 From A-Level Maths [3 marks]

Find the equation of the perpendicular bisector of the points $(1, 2)$ and $(5, 4)$.

Answer .....

A3 Last lesson [2 marks]

Write $4\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ in the form $a + bi$.

Answer .....

A4 Needed today [2 marks]

Find the distance between the points representing $5 + 2i$ and $1 - i$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Circles and perpendicular bisectors

EXAMPLE 1 · I DO

Sketch the locus $|z-(3+2i)|=4$ on an Argand diagram. Find its cartesian equation, and write the locus in set notation.

Hint $|z-a|$ is the distance from z to the point a.

YOUR TURN · WE DO

B1 Sketch the locus $|z+2-5i|=3$. State its centre and radius, and find its cartesian equation. [4 marks]

Answer .....

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EXAMPLE 2 · I DO

Sketch the locus $|z - 2| = |z - 4i|$ and find its cartesian equation.

Hint The points are the same distance from 2 and from $4i$.

YOUR TURN · WE DO

B2 Sketch the locus $|z + 1| = |z - 3 - 2i|$ and find its cartesian equation in the form $y = mx + c$. [4 marks]

Answer .....

PRACTICE · YOU DO

C1 Sam says that the locus $|z + 4 - 3i| = 2$ is a circle with centre $4 - 3i$. Explain Sam's mistake. Write down the correct centre, write the locus in set notation, and find its cartesian equation. [4 marks]

Answer .....

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C2 Sketch the locus $|z - 3i| = |z - 4 + i|$, marking the points $3i$ and $4 - i$. Find its cartesian equation. Show that the line is perpendicular to the line joining $3i$ and $4 - i$, and passes through their midpoint. [5 marks]

Answer .....

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PART 2 Half-lines and set notation

EXAMPLE 3 · I DO

Sketch the locus $\arg(z - 1 - i) = \frac{\pi}{4}$ and find its cartesian equation.

Hint It starts at the point $1 + i$ and goes off at the angle $\frac{\pi}{4}$ to the positive real direction.

YOUR TURN · WE DO

B3 Sketch the locus $\arg(z + 2) = \frac{2\pi}{3}$ and find its cartesian equation, stating the values of x for which it holds. [4 marks]

Answer .....

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C3 Sketch the locus $\arg(z - 1 + 3i) = \dfrac{3\pi}{4}$ and find its cartesian equation, stating the values of x for which it holds. [4 marks]

Answer .....

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C4 Write each locus in set notation. (i) The circle with centre $-1 + 4i$ that passes through O. (ii) The half-line from the point $2i$ that makes an angle $-\dfrac{\pi}{6}$ with the positive real direction. (iii) The points the same distance from $1 + i$ as from $5 + i$. Give the cartesian equation of the locus in (iii). [6 marks]

Answer .....

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PART 3 Where two loci meet

EXAMPLE 4 · I DO

Find the points where the circle $|z - 1 - 2i| = 5$ meets the line $\mathrm{Re}(z) = 4$.

Hint Put $x = 4$ into the cartesian equation of the circle.

YOUR TURN · WE DO

B4 Find the points where the circle $|z + 3 - i| = 5$ meets the line $\mathrm{Im}(z) = -3$. [4 marks]

Answer .....

PRACTICE · YOU DO

C5 Find the points where the locus $|z - 2| = |z - 2i|$ meets the circle $|z| = 4$. Give your answers exactly, as complex numbers. [5 marks]

Answer .....

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C6 Find the point where the half-line $\arg(z+1) = \dfrac{\pi}{4}$ meets the circle $|z-3| = \sqrt{26}$. Explain why there is only one point. [5 marks]

Answer .....

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PART 4 Least and greatest distance on a circle

EXAMPLE 5 · I DO

z lies on the circle $|z - 6 - 8i| = 4$. Find the least and greatest values of $|z|$, and the points where they happen.

Hint $|z|$ is the distance from O. The nearest and furthest points are on the line through O and the centre.

YOUR TURN · WE DO

B5 z lies on the circle $|z + 5 - 12i| = 6$. Find the least and greatest values of $|z|$, and the point where $|z|$ is least. [5 marks]

Answer .....

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C7 z lies on the circle $|z - 15 + 20i| = 5$. Find the least and greatest values of $|z|$, and the point where $|z|$ is greatest. [5 marks]

Answer .....

C8 z lies on the circle $|z + 3 + 4i| = 2$. Find the least and greatest values of $|z - 3 - 4i|$, and the point on the circle nearest to $3 + 4i$. [6 marks]

Answer .....

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D1

> Alex says that L is the whole line $y = 3 - x$.

In this question you must show detailed reasoning. The locus L is $\arg(z - 3i) = -\frac{\pi}{4}$.

(a) Explain why Alex is wrong, and describe L exactly. [2 marks]

(b) Find the point where L meets the real axis. [2 marks]

Answer .....

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(c) Find, exactly, the point where L meets the circle $|z - 4 - 3i| = 5$. [5 marks]

Answer .....

Total for D1: 9 marks

D2 The complex number z lies on the circle $|z - 8 - 6i| = 6$.

(a) Show that the circle touches the real axis. [2 marks]

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(b) Find the least value of $|z|$, and the point where it happens. [3 marks]

Answer .....

(c) Find the greatest value of $\arg z$, to 3 significant figures, and the point where it happens. [5 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The circle C is $|z - 6 - 2i| = r$, where $r > 0$, and the half-line L is $\arg(z - 2i) = \frac{\pi}{4}$. Find the value of r for which L is a tangent to C, and the point where they touch. Find the set of values of r for which L meets C at two points. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write L as a cartesian equation with a condition on x.

Hint 2 Put it into the equation of the circle to get a quadratic in x. A tangent gives a repeated root.

Hint 3 For two points, both roots must be on the half-line, so both must be positive. At $r = 6$ one root is $x = 0$, the point $2i$, which is not on L.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch the circle $|z - a| = r$, the perpendicular bisector $|z - a| = |z - b|$ and the half-line $\arg(z - a) = \theta$. ○ ○ ○

I can find the cartesian equation of a locus and write a locus in set notation. ○ ○ ○

I can find where two loci meet and give the answer as a complex number. ○ ○ ○

I can find the least and greatest $|z|$, and the greatest argument, for z on a circle. ○ ○ ○

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