Matrix arithmetic and multiplication

89 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-matrix-arithmetic/

Date

A Do Now

Quick questions to start.

A1 From A-Level Maths [2 marks]

Solve $2a + b = 7$ and $a - b = 2$.

Answer .....

A2 Needed today [2 marks]

Expand and simplify $(a + 2)^2 - (a - 1)(a + 3)$.

Answer .....

A3 From chapter 1 [2 marks]

Write $(2 + i)(3 - i)$ in the form $a + bi$.

Answer .....

A4 Needed today [2 marks]

Solve $a^2 = 4a$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Order, adding and multiplying by a number

EXAMPLE 1 · I DO

$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 5 & -2 \\ 3 & 0 & 2 \end{pmatrix}$. State the order of $\mathbf{A}$, and find $2\mathbf{A} - 3\mathbf{B}$.

Hint The order is rows by columns. Multiply every entry by the scalar, then subtract entry by entry.

YOUR TURN · WE DO

B1 [4 marks]

$\mathbf{P} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \\ 0 & 5 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -1 & 2 \\ 4 & 0 \\ 2 & -3 \end{pmatrix}$.

State the order of $\mathbf{P}$, find $3\mathbf{P} + 2\mathbf{Q}$, and write down $\mathbf{Q}^{\mathrm{T}}$.

Answer .....

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C1 Find the values of x, y and z for which [4 marks]

$$2\begin{pmatrix} x & 1 \\ y & 3 \end{pmatrix} - \begin{pmatrix} 4 & y \\ 1 & z \end{pmatrix} = \begin{pmatrix} z & x-3 \\ 3 & x+1 \end{pmatrix}.$$

Answer .....

C2 Find the matrix $\mathbf{X}$ for which $3\mathbf{X} - \begin{pmatrix} 2 & -1 \\ 4 & 0 \end{pmatrix} = \mathbf{X} + \begin{pmatrix} 6 & 3 \\ -2 & 8 \end{pmatrix}$. [4 marks]

Hence find $\mathbf{X}^2$.

Answer .....

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PART 2 Multiplying matrices

EXAMPLE 2 · I DO

$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}$.

Find $\mathbf{AB}$ and $\mathbf{BA}$.

Hint Each entry of $\mathbf{AB}$ is a row of $\mathbf{A}$ times a column of $\mathbf{B}$: first times first, plus second times second.

YOUR TURN · WE DO

B2 $\mathbf{C} = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}$. Find $\mathbf{CD}$ and $\mathbf{DC}$. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

$\mathbf{P} = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 4 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. One of $\mathbf{PQ}$ and $\mathbf{QP}$ exists. Say which, with a reason, and find it.

Hint Write the orders side by side, $(2 \times 3)(3 \times 1)$. The two middle numbers must be equal, and the outside numbers give the order of the product.

YOUR TURN · WE DO

B3 [4 marks]

$\mathbf{R} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 4 \end{pmatrix}$ and $\mathbf{S} = \begin{pmatrix} 5 & -2 & 1 \end{pmatrix}$. One of $\mathbf{RS}$ and $\mathbf{SR}$ exists. Say which, with a reason, and find it.

Answer .....

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C3 $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$. Sam writes: "$\mathbf{AB} \neq \mathbf{BA}$, because matrix multiplication is not commutative." Find $\mathbf{AB}$ and $\mathbf{BA}$, and explain Sam's mistake. [4 marks]

Answer .....

C4 $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$. Show that $(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$, and that $(\mathbf{AB})^{\mathrm{T}} \neq \mathbf{A}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}$. [5 marks]

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C5 $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & -1 \end{pmatrix}$. Find $\mathbf{C}(\mathbf{AB})$ and $(\mathbf{CA})\mathbf{B}$. Name the property of matrix multiplication that your answers show. [6 marks]

Answer .....

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PART 3 Powers and matrices that commute

EXAMPLE 4 · I DO

$\mathbf{M} = \begin{pmatrix} a & 2 \\ -1 & 3 \end{pmatrix}$. Find $\mathbf{M}^2$ in terms of a. Find the value of a for which $\mathbf{M}^2$ is a multiple of $\mathbf{I}$, and write down $\mathbf{M}^2$ for this value.

Hint A multiple of $\mathbf{I}$ has 0 in both off-diagonal places and equal diagonal entries. Use one equation to find a, then check every entry.

YOUR TURN · WE DO

B4 $\mathbf{N} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$. Find $\mathbf{N}^2$ in terms of a. Find the value of a for which $\mathbf{N}^2$ is a multiple of $\mathbf{I}$, and write down $\mathbf{N}^2$ for this value. [5 marks]

Answer .....

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EXAMPLE 5 · I DO

$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Find every matrix $\mathbf{B} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ for which $\mathbf{AB} = \mathbf{BA}$.

Hint Work out both products in terms of p, q, r, s and set the four pairs of entries equal.

YOUR TURN · WE DO

B5 $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$. Find every matrix $\mathbf{B} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ for which $\mathbf{AB} = \mathbf{BA}$. [5 marks]

Answer .....

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C6 $\mathbf{M} = \begin{pmatrix} 2 & k \\ 3 & -2 \end{pmatrix}$, where k is a constant. Find $\mathbf{M}^2$ in terms of k, and show that $\mathbf{M}^2$ is a multiple of $\mathbf{I}$ for every value of k. Find the value of k for which $\mathbf{M}^2 = 13\mathbf{I}$. [5 marks]

Answer .....

C7 $\mathbf{M} = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$, where a and b are real.

(a) Show that $\mathbf{M}^2 = (a^2 + b^2)\mathbf{I} + 2ab\mathbf{J}$, where $\mathbf{J} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. [2 marks]

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(b) Find every matrix $\mathbf{M}$ of this form for which $\mathbf{M}^2 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}$. [6 marks]

Answer .....

Total for C7: 8 marks

D Exam-style questions

OCR exam style

D1 Alex writes: "$\mathbf{P}$ is 3×2 and $\mathbf{Q}$ is 2×3, so $\mathbf{PQ}$ and $\mathbf{QP}$ are both 3×3 matrices."

$\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 3 & -2 \end{pmatrix}$.

(a) Explain Alex's mistake, and give the order of each product. [2 marks]

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(b) Find $\mathbf{QP}$. [3 marks]

Answer .....

(c) Find $\mathbf{PQ}$. [3 marks]

Answer .....

Total for D1: 8 marks

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D2 $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ -1 & k \end{pmatrix}$, where k is a real constant. $\mathbf{M}^3 = \begin{pmatrix} p & 6 \\ -6 & q \end{pmatrix}$. [7 marks]

Find every possible set of values of k, p and q.

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find every real 2×2 matrix $\mathbf{X}$ for which $\mathbf{X}^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\mathbf{X}$ always commutes with $\mathbf{X}^2$. Why?

Hint 2 So $\mathbf{X}$ commutes with $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Use this to cut down the form of $\mathbf{X}$.

Hint 3 Square your simpler form of $\mathbf{X}$ and compare entries. You should find two matrices.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can say when two matrices are conformable for multiplication, and give the order of the product.	○ ○ ○
I can add, subtract, scale and multiply matrices, including complex matrices.	○ ○ ○
I can find a power of a matrix in terms of a letter, and solve by equating every entry.	○ ○ ○
I can explain, with a counter-example or a calculation, that matrix multiplication is associative but not commutative.	○ ○ ○

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