Solving simultaneous equations with matrices

98 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-matrix-equations/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the determinant of $\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 1 & 4 \end{pmatrix}$.

Answer .....

A2 Last lesson [2 marks]

Find the inverse of $\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$.

Answer .....

A3 Needed today [2 marks]

Write the equations

$$x + 2y - z = 3$$
$$2x - y = 1$$
$$y + 3z = 4$$

as a single matrix equation.

Answer .....

A4 From A-Level Maths [2 marks]

Solve $2x + 3y = 1$ and $5x + 4y = 13$.

Answer .....

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B Examples and practice

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

PART 1 Two equations with an inverse matrix

EXAMPLE 1 · I DO

Use a matrix method to solve $3x + 2y = 4$ and $5x + 4y = 6$.

Hint Write $\mathbf{Mx} = \mathbf{b}$. Then $\mathbf{x} = \mathbf{M}^{-1}\mathbf{b}$: multiply by the inverse on the LEFT.

YOUR TURN · WE DO

B1 Use a matrix method to solve $4x - 3y = 5$ and $3x - 2y = 4$. Show the inverse matrix. [4 marks]

Answer .....

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EXAMPLE 2 · I DO

The equations $3x + ky = 6$ and $kx + 12y = 12$ have one solution. Find the values k cannot take, and the solution in terms of k.

Hint One solution exactly when the determinant is not zero. Find the inverse in terms of k and look for a factor that cancels.

YOUR TURN · WE DO

B2 The equations $2x + ky = 3$ and $kx + 8y = 6$ have one solution. Find the values k cannot take, and the solution in terms of k. Then say what happens when $k = 4$ and when $k = -4$. [6 marks]

Answer .....

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C1 The equations $6x - 4y = 2$ and $9x - 6y = k$ are given. Show that they do not have a single solution, find the value of k for which they have solutions, and describe those solutions. [4 marks]

Answer .....

C2 $\mathbf{M} = \begin{pmatrix} k & 2 \\ 3 & k-1 \end{pmatrix}$, where k is a constant.

(a) Find the values of k for which $\mathbf{M}$ is singular. [2 marks]

Answer .....

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(b) Hence, for the other values of k, use $\mathbf{M}^{-1}$ to solve $kx + 2y = 1$ and $3x + (k-1)y = 0$, giving x and y in terms of k. [4 marks]

Answer .....

(c) Determine whether there is a value of k for which the solution has $x = y$. [3 marks]

Answer .....

Total for C2: 9 marks

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PART 2 Three planes that meet at one point

EXAMPLE 3 · I DO

Solve

$$x + y + z = 6$$
$$2x - y + z = 3$$
$$x + 2y - z = 2$$

using an inverse matrix, and say what the solution means for the three planes.

Hint The determinant is not zero, so there is exactly one solution: the three planes meet at a single point. Find the inverse by cofactors and show it.

YOUR TURN · WE DO

B3 In this question you must show detailed reasoning. The planes [5 marks]

$$2x + y = 4$$
$$x + y + z = 2$$
$$y + 3z = -1$$

are given. Show that they meet at a single point, and use the inverse matrix to find it.

Answer .....

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C3 Sam writes the equations [6 marks]

$$x + 2y = 4$$
$$3x - z = 6$$
$$y + 4z = 1$$

as $\begin{pmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$. Explain Sam's mistake. Then solve the equations with a matrix method.

Answer .....

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C4 A group of 40 people buys tickets for a show. Adult tickets cost £6, child tickets £3 and senior tickets £4, and the group pays £180. There are 5 more children than seniors. Write the facts as a matrix equation, and use the inverse matrix to find how many of each ticket are bought. [6 marks]

Answer .....

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PART 3 When the determinant is zero

EXAMPLE 4 · I DO

Show that the planes

$$x + y + z = 2 \quad [1]$$
$$x - y + 2z = 1 \quad [2]$$
$$2x + 3z = 3 \quad [3]$$

have a line in common.

Hint Find the determinant first. If it is zero, look for numbers p and q with $p[1] + q[2]$ equal to $[3]$, left-hand sides AND right-hand sides.

YOUR TURN · WE DO

B4 Show that the planes [4 marks]

$$2x - y + z = 3 \quad [1]$$
$$x + y - z = 0 \quad [2]$$
$$5x - y + z = 6 \quad [3]$$

form a sheaf.

Answer .....

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EXAMPLE 5 · I DO

The planes

$$x + y + z = 2 \quad [1]$$
$$x - y + 2z = 1 \quad [2]$$
$$2x + 3z = 7 \quad [3]$$

have no point in common. Show this and describe how they are arranged. Then describe $x + 2y - z = 3$, $2x + 4y - 2z = 5$ and $x + y + z = 1$.

Hint The left-hand sides are those of the last example. If no two planes are parallel and there is no common point, the planes form a prism. Two equations whose left-hand sides are multiples but whose right-hand sides are not give parallel planes.

YOUR TURN · WE DO

B5 Show that the planes [5 marks]

$$x - 2y + z = 1 \quad [1]$$
$$2x + y - z = 3 \quad [2]$$
$$3x - y = 5 \quad [3]$$

form a prism.

Answer .....

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C5 For each set of planes, say whether they meet at a single point, in a line (a sheaf), not at all because two planes are parallel, or not at all as a prism. Give a reason each time. (i) $x + y + z = 1$, $2x + 2y + 2z = 5$, $x - y + z = 0$. (ii) $x + y = 3$, $y + z = 5$, $x + z = 4$. (iii) $x + 2y - z = 1$, $2x + y + z = 4$, $x - y + 2z = 2$ [6 marks]

Answer .....

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C6

$$x + 2y - z = 1 \quad [1]$$
$$3x + y + kz = 4 \quad [2]$$
$$x - 3y + 4z = c \quad [3]$$

where k and c are constants.

(a) Find the value of k for which the equations do not have a single solution. [3 marks]

Answer .....

(b) With this value of k, find the value of c for which the equations are consistent. Hence describe the planes for that value of c. [4 marks]

Answer .....

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(c) With this value of k and $c \neq 2$, describe the planes, with a reason. [2 marks]

Answer .....

Total for C6: 9 marks

D Exam-style questions

OCR exam style

D1

> Alex writes: "The determinant of the matrix of coefficients is zero, so the three planes have no point in common."

The planes Π_1, Π_2, Π_3 have equations

$$x + 2y - z = 3$$
$$2x - y + 3z = 1$$
$$4x + 3y + z = 7$$

(a) Show that the determinant is zero. [2 marks]

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(b) Show that Alex is wrong, and describe how the planes are arranged. [3 marks]

Answer .....

(c) Find the point on all three planes with $z = 0$, and the point on all three planes with $z = 1$. [3 marks]

Answer .....

Total for D1: 8 marks

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D2 $\mathbf{A} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

(a) Find $\det \mathbf{A}$, and explain what it tells you about the planes $2x + y - z = p$, $x + 3y + 2z = q$, $x - y + z = r$ for any constants p, q, r. [2 marks]

Answer .....

(b) Find $\mathbf{A}^{-1}$. [4 marks]

Answer .....

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(c) Hence find the point where the planes $2x + y - z = -1$, $x + 3y + 2z = 2$ and $x - y + z = 4$ meet. [3 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The three planes $x + y + z = 1$, $2x + ay + 2z = b$ and $x + 2y + az = 3$ are given, where a and b are constants. Find every value of a, and the matching values of b, for which the planes do not meet at a single point. In each case say how the planes are arranged and whether they have any point in common. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 When is there exactly one point? Find the determinant of the coefficients in terms of a.

Hint 2 For each value of a that makes it zero, write the three left-hand sides out and look for one that is a multiple, or a combination, of the others.

Hint 3 Once you have a combination for the left-hand sides, do the same to the right-hand sides. Matching right-hand sides means the planes share points.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write simultaneous equations as a matrix equation and solve them with the inverse matrix.	○ ○ ○
I can say that a non-zero determinant means the three planes meet at one point.	○ ○ ○
I can show three equations are consistent with a labelled combination such as [1] + [2] = [3].	○ ○ ○
I can tell a sheaf, a prism and parallel planes apart and give the reason.	○ ○ ○

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