# Banked tracks and motion on a rough surface

86 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-banked/

Date



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## A Do Now

Quick questions to start.

**A1** From A-Level Maths [2 marks]

A force of $40\ \mathrm{N}$ acts at right angles to a slope inclined at $25^\circ$ to the horizontal. State the angle the force makes with the vertical, and find its horizontal component.

Answer .....

**A2** Last lesson [2 marks]

A particle of mass $0.2\ \mathrm{kg}$ moves as a conical pendulum with the string at $45^\circ$ to the vertical. Find the tension.

Answer .....

**A3** Needed today [2 marks]

A car of mass $800\ \mathrm{kg}$ goes round a horizontal circle of radius $60\ \mathrm{m}$ at $15\ \mathrm{m\,s^{-1}}$. Find the resultant force on the car, and state its direction.

Answer .....

**A4** Needed today [1 mark]

Change $72\ \mathrm{km\,h^{-1}}$ to $\mathrm{m\,s^{-1}}$.

Answer .....

## B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).



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## PART 1 Level bends and smooth banked tracks

**EXAMPLE 1 · I DO**

A car of mass $1200\ \text{kg}$ goes round a level bend that is an arc of a circle of radius $50\ \text{m}$. The sideways friction force on the car can be at most $7000\ \text{N}$. Find the greatest speed at which it can go round the bend.

**Hint** On a level road the only horizontal force is friction, so friction $= \dfrac{mv^2}{r}$. The weight is vertical: it does not appear in this equation.

**YOUR TURN · WE DO**

**B1** A lorry of mass $9000\ \text{kg}$ goes round a level bend of radius $80\ \text{m}$ at $15\ \text{m s}^{-1}$. Find the friction force on the lorry. The friction can be at most $30\,000\ \text{N}$: find the greatest safe speed. [4 marks]

Answer .....


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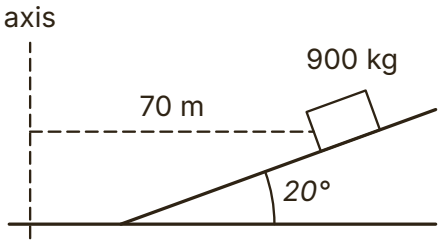
**EXAMPLE 2 · I DO**

A bend on a smooth track is a horizontal circle of radius $70\ \mathrm{m}$, banked at $20^\circ$ to the horizontal. Find the speed at which a car of mass $900\ \mathrm{kg}$ can go round it, and the normal contact force on the car.




**Hint** The car does not slide down the slope, and its acceleration is horizontal, towards the centre. So resolve vertically: $R\cos\alpha = mg$, not $R = mg\cos\alpha$.

Horizontally: $R\sin\alpha = \dfrac{mv^2}{r}$.

**YOUR TURN · WE DO**

**B2** A bend of radius $150\ \mathrm{m}$ on a smooth track is banked so that cars can go round it at $25\ \mathrm{m\,s^{-1}}$. Find the angle of banking, and the normal contact force on a car of mass $1100\ \mathrm{kg}$ going round at this speed. [4 marks]

Answer .....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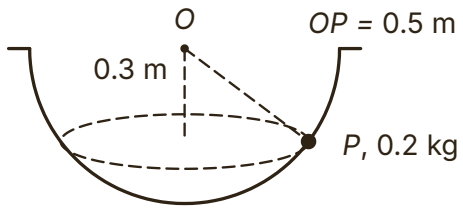
**C1** A particle $P$ of mass $0.2\ \text{kg}$ moves in a horizontal circle on the inside of a smooth hemispherical bowl of radius $0.5\ \text{m}$. The rim of the bowl is horizontal, and the circle is $0.3\ \text{m}$ below the rim. Find the normal contact force and the angular speed. [4 marks]




Answer .....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**C2** A bend of radius $50\ \text{m}$ on a smooth track is banked at $18°$. A car of mass $1000\ \text{kg}$ goes round it with no tendency to slip. Sam writes: "The car does not move into the slope, so $R = mg\cos 18° = 9320\ \text{N}$." Explain Sam's mistake. Find the correct contact force and the speed of the car. [5 marks]

Answer .....


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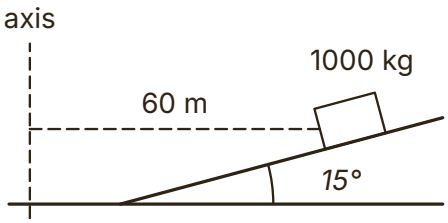
## PART 2 Friction on a banked track, up or down the slope

**EXAMPLE 3 · I DO**

A bend of radius $60\ \text{m}$ is banked at $15°$. A car of mass $1000\ \text{kg}$ goes round it at $20\ \text{m s}^{-1}$. Find the friction force and the normal contact force on the car.




**Hint** This is faster than the speed with no friction, so the car tends to slide up the slope and friction acts down it. Both $R$ and $F$ have a vertical and a horizontal component: $R$ makes $15°$ with the vertical, $F$ makes $15°$ with the horizontal.

**YOUR TURN · WE DO**

**B3** A bend of radius $90\ \text{m}$ is banked at $12°$. A car of mass $1500\ \text{kg}$ goes round it at $24\ \text{m s}^{-1}$. Find the friction force on the car, stating its direction, and the normal contact force. [4 marks]

Answer .....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**EXAMPLE 4 · I DO**

A bend of radius $40\ \mathrm{m}$ is banked at $25°$. A car of mass $800\ \mathrm{kg}$ goes round it slowly, at $8\ \mathrm{m\,s^{-1}}$. Find the friction force, stating its direction, and the normal contact force.

**Hint** If you are not sure which way friction acts, take it down the slope: a negative answer means it acts up the slope. Here the car is slow, so it tends to slide down.

**YOUR TURN · WE DO**

**B4** A cyclist and bicycle, of total mass $80\ \mathrm{kg}$, ride round a track banked at $30°$ in a horizontal circle of radius $25\ \mathrm{m}$, at $6\ \mathrm{m\,s^{-1}}$. Find the friction force, stating its direction, and the normal contact force. [4 marks]

Answer .....


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**C3** A bend of radius $120\ \mathrm{m}$ is banked at $8°$. A car of mass $900\ \mathrm{kg}$ goes round it at $10\ \mathrm{m\,s^{-1}}$. Find the speed at which no friction would be needed, and use it to decide which way the friction on the car acts. Find the friction force. [4 marks]

Answer .....

**C4** A railway curve has radius $400\ \mathrm{m}$. It is banked so that a train going round it at $20\ \mathrm{m\,s^{-1}}$ puts no sideways force on the rails. Find the angle of banking. A train of mass $50\,000\ \mathrm{kg}$ goes round the curve at $30\ \mathrm{m\,s^{-1}}$. Find the sideways force from the rails on the train, parallel to the track surface. [5 marks]

Answer .....


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## PART 3 The greatest and least speeds on a rough bend

**EXAMPLE 5 · I DO**

A bend of radius $80\ \text{m}$ is banked at $10^\circ$. The friction force on a car of mass $1000\ \text{kg}$ can be at most $4000\ \text{N}$. Find the greatest speed at which the car can go round the bend. Show that the car can go round it at any lower speed, however slow.

**Hint** At the greatest speed the car is about to slide up, so friction is $4000\ \text{N}$ down the slope. For the least speed, put $4000\ \text{N}$ up the slope and see whether $\dfrac{mv^2}{r}$ comes out positive.

**YOUR TURN · WE DO**

**B5** A bend of radius $100\ \text{m}$ is banked at $15^\circ$. The friction force on a car of mass $1200\ \text{kg}$ can be at most $3000\ \text{N}$. Find the greatest and the least speed at which the car can go round the bend without sliding. [5 marks]

Answer .....


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**C5** A cycle track has a bend of radius $30\ \text{m}$, banked at $40°$. A cyclist and bicycle have total mass $85\ \text{kg}$. The sideways friction force on the tyres can be at most $300\ \text{N}$. Find the greatest speed at which the cyclist can go round the bend. [3 marks]

Answer .....


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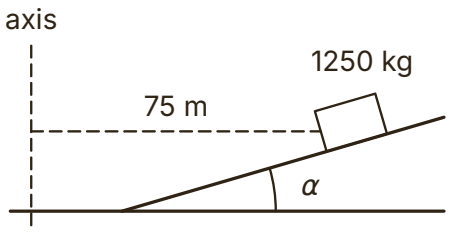
**C6** A bend of radius $75\ \text{m}$ is banked at an angle $\alpha$ to the horizontal, where $\tan\alpha = \frac{7}{24}$. A car of mass $1250\ \text{kg}$ goes round the bend.




**(a)** Find the speed at which the car can go round the bend with no friction force. [3 marks]

Answer .....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**(b)** The car goes round the bend at $25\ \mathrm{m\,s^{-1}}$. Find the friction force and the normal contact force, using the exact values $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$. [4 marks]

Answer .....

**(c)** The friction force can be at most $5000\ \mathrm{N}$. Find the greatest speed at which the car can go round the bend. [3 marks]

Answer .....

**Total for C6: 10 marks**


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**D1**

> Alex says: “A heavier car has to go faster to get round this bend without slipping.”

A car of mass $m$ goes round a smooth bend of radius $r$, banked at angle $\alpha$, with no tendency to slip.

**(a)** Show that the speed of the car is $\sqrt{rg\tan\alpha}$. [3 marks]

**(b)** Explain why Alex is wrong. [2 marks]

**(c)** Find the speed when $r = 60\ \text{m}$ and $\alpha = 22°$. [2 marks]

Answer .....


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**(d)** Find the normal contact force on a car of mass $1400\ \text{kg}$ on this bend. [2 marks]

Answer .....

**Total for D1: 9 marks**

**D2** A light aircraft of mass $1500\ \text{kg}$ turns in a horizontal circle of radius $400\ \text{m}$ at a constant $60\ \text{m}\,\text{s}^{-1}$. The lift force on the aircraft acts at right angles to its wings.

**(a)** Explain why the aircraft must bank its wings to turn. [2 marks]

**(b)** Find the angle at which the wings are banked. [3 marks]

Answer .....


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**(c)** Find the lift force. [2 marks]

Answer .....

**(d)** Find the time the aircraft takes to turn through $180°$. [3 marks]

Answer .....

**Total for D2: 10 marks**


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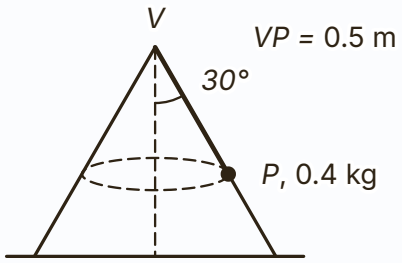
## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** A smooth solid cone is fixed with its axis vertical and its vertex $V$ at the top. Its surface makes an angle of $30^\circ$ with the axis. A particle $P$ of mass $0.4\ \text{kg}$ is joined to $V$ by a light inextensible string of length $0.5\ \text{m}$, which lies along the surface. $P$ moves in a horizontal circle on the outside of the cone with angular speed $\omega$. Find the tension and the normal contact force when $\omega = 3\ \text{rad}\,\text{s}^{-1}$. Find the greatest value of $\omega$ for which $P$ stays on the surface. [8 marks]




Answer .....



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Draw the forces at $P$: the weight, the tension along the string towards $V$, and the contact force at right angles to the surface, pushing outwards.

**Hint 2** Resolve vertically and horizontally, not along the surface: the acceleration $r\omega^2$ is horizontal, towards the axis.

**Hint 3** As $\omega$ grows, the contact force gets smaller. $P$ is about to leave the surface when $R = 0$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the speed on a smooth banked track, using $R\cos\alpha = mg$ and $R\sin\alpha = \dfrac{mv^2}{r}$. ○ ○ ○

I can find the friction and the contact force on a rough banked track, and say which way the friction acts. ○ ○ ○

I can find the greatest and least speeds when the friction is at most a given force. ○ ○ ○

I can apply the same method to an aircraft, a bowl or a cone, and say when a particle leaves a surface. ○ ○ ○



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