Finding unknown indices and forming a model

94 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-dim-indices/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Write down the dimensions of pressure and of density.

Answer .....

A2 Needed today [2 marks]

Solve $\alpha + \beta = 0$ and $2\alpha - \beta = 3$.

Answer .....

A3 Last lesson [2 marks]

A stone is dropped from a height h. Show that the formula $t = \sqrt{\frac{h}{g}} + h$ for the time it takes to fall cannot be correct.

Answer .....

A4 Needed today [2 marks]

Simplify $(\mathrm{LT}^{-2})^{\frac{1}{2}} \times \mathrm{L}^{\frac{1}{2}}$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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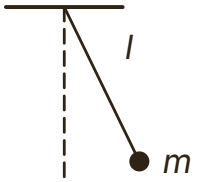
PART 1 Finding three unknown powers

EXAMPLE 1 · I DO

The period t of a simple pendulum is thought to depend on the mass m of the bob, the length l of the string and g. Use dimensional analysis to find t in the form $t = km^{\alpha}l^{\beta}g^{\gamma}$, where k is a dimensionless constant.



Hint Write the formula with k in it. Put in the dimensions of every quantity, multiply every power in a bracket, and write one equation each for M, L and T. Do not use a formula you already know: the method must come from the dimensions.

YOUR TURN · WE DO

B1 A ball of mass m is dropped from rest and falls a height h. Its speed is thought to be $v = km^{\alpha}h^{\beta}g^{\gamma}$ where k is a dimensionless constant. Use dimensional analysis to find α, β and γ, and write the formula for v. [5 marks]

Answer .....

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EXAMPLE 2 · I DO

The least speed v with which a rocket must leave a planet to escape it depends on the planet's mass M, its radius R and the constant G, where $[G] = \mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$. Find v in the form $v = kG^{\alpha}M^{\beta}R^{\gamma}$.

Hint G is a named constant, so it is not dimensionless: its dimensions go into the equation like any other quantity.

YOUR TURN · WE DO

B2 A small drop of liquid wobbles with frequency f. Model $f = k\sigma^{\alpha}\rho^{\beta}r^{\gamma}$, where σ is the surface tension of the liquid, with $[\sigma] = \mathrm{MT}^{-2}$, ρ is its density and r is the radius of the drop. Find α, β and γ, and write the formula for f. [5 marks]

Answer .....

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C1 The speed of sound v in a gas is thought to depend on the pressure p of the gas and its density ρ. Use dimensional analysis to find v in the form $v = kp^{\alpha}\rho^{\beta}$, where k is a dimensionless constant. [4 marks]

Answer .....

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C2 The speed v of a wave along a stretched string depends on the tension F in the string and its mass per unit length μ, where $[\mu] = \mathrm{ML}^{-1}$. Sam writes: [5 marks]

$$v = F^{\alpha}\mu^{\beta}, \qquad \mathrm{LT}^{-1} = (\mathrm{MLT}^{-2})^{\alpha}(\mathrm{ML}^{-1})^{\beta} = \mathrm{M}^{\alpha+\beta}\mathrm{L}^{\alpha-\beta}\mathrm{T}^{-2}.$$

Describe two mistakes in Sam's work, then find the correct formula for v.

Answer .....

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PART 2 Using the formula to scale a quantity

EXAMPLE 3 · I DO

Use $t = k\sqrt{\dfrac{l}{g}}$ for the pendulum. (i) What does the power of m being 0 tell you? (ii) The string is made 9 times as long. What happens to the period? (iii) How must l change to halve the period?

Hint A power of 0 means the quantity does not appear. For a scale factor, put the factor into the formula: t is proportional to $l^{\frac{1}{2}}$.

YOUR TURN · WE DO

B3 Use $f = k\sqrt{\dfrac{\sigma}{\rho r^3}}$ for the drop in B2. (i) A drop of the same liquid has 4 times the radius. Find the factor by which its frequency changes. (ii) A drop of the same size, in a liquid with the same surface tension, wobbles 3 times as fast. Find the factor by which the density has changed. [4 marks]

Answer .....

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C3 Dimensional analysis gives the time for a stone to fall a height h from rest as $t = k\sqrt{\dfrac{h}{g}}$. On Mars $g = 3.7\ \mathrm{m\,s^{-2}}$. Find the height on Mars that a stone falls in the same time as it takes to fall $2.0\ \mathrm{m}$ on Earth. On another planet, g is a quarter of its value on Earth. Find the factor by which the fall time changes when the height is also made 4 times as great. [4 marks]

Answer .....

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C4 [4 marks]

The speed of a wave along a stretched string is $v = k\sqrt{\dfrac{F}{\mu}}$, where F is the tension and μ is the mass per unit length. (i) The tension is increased by 44% and μ is unchanged. Find the percentage increase in the speed. (ii) The tension is kept the same. Find the factor by which μ must change to halve the speed.

Answer .....

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PART 3 Finding unknown constants

EXAMPLE 4 · I DO

The power P of a wind turbine is modelled as $P = k\rho^{\alpha}v^{\beta}l^{\gamma}\omega^{\delta}$, where ρ is the density of the air, v the wind speed, l the blade length and ω the angular speed of the blades. (i) Explain why dimensional analysis cannot find all four powers. (ii) Find α, β and γ in terms of δ. (iii) Tests show that P does not depend on ω. Find the formula for P.

Hint There is one index equation for each of M, L and T. Solve them, leaving δ as it is.

YOUR TURN · WE DO

B4 [6 marks]

Water flows along a narrow pipe of radius r. The pressure falls by s per unit length of pipe, so $[s] = \mathrm{ML}^{-2}\mathrm{T}^{-2}$. The volume flowing per second, Q, is modelled as $Q = ks^{\alpha}r^{\beta}\eta^{\gamma}\rho^{\delta}$, where $[\eta] = \mathrm{ML}^{-1}\mathrm{T}^{-1}$ and ρ is the density. Find α, β and γ in terms of δ. Experiments show that Q does not depend on ρ. Find the formula for Q.

Answer .....

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EXAMPLE 5 · I DO

A particle moves on a line with constant acceleration a, starting with speed u. A model for its displacement after time t is $s = k_1 u^{\alpha} t^{\beta} + k_2 a^{\gamma} t^{\delta}$. Find the four powers. When $a = 0$ and $u = 4\ \mathrm{m\,s^{-1}}$, $s = 40\ \mathrm{m}$ after $10\ \mathrm{s}$. From rest with $a = 3\ \mathrm{m\,s^{-2}}$, $s = 24\ \mathrm{m}$ after $4\ \mathrm{s}$. Use these two results to find the constants.

Hint Each term on its own must have the dimensions of s, so do the index equations term by term. Then use each experiment to find one constant: in each, one term is zero.

YOUR TURN · WE DO

B5 A body of mass m moving at speed u is pushed a distance s by a constant force F. A model for its final kinetic energy is $E = k_1 m^{\alpha} u^{\beta} + k_2 F^{\gamma} s^{\delta}$. Find the four powers. With no force, a body of mass $2\ \mathrm{kg}$ moving at $3\ \mathrm{m\,s^{-1}}$ has $E = 9\ \mathrm{J}$. Starting from rest, a force of $5\ \mathrm{N}$ acting over $4\ \mathrm{m}$ gives $E = 20\ \mathrm{J}$. Find k_1 and k_2. [6 marks]

Answer .....

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C5 A golf ball of mass m is hit from level ground with speed u, at an angle θ to the ground. The distance it travels before landing is modelled as $R = ku^{\alpha}g^{\beta}m^{\gamma}\theta^{\delta}$. Explain why dimensional analysis cannot find δ. Find α, β and γ, and write the formula for R. [5 marks]

Answer .....

C6 A ball runs along a smooth track. Its speed is u at a point A, and its speed is v at a point B a height h below A. A model is $v^2 = k_1u^{\alpha} + k_2g^{\beta}h^{\gamma}$, where k_1 and k_2 are dimensionless constants.

(a) Explain why both terms on the right must have the same dimensions as v^2. [2 marks]

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(b) Find α, β and γ. [4 marks]

Answer .....

(c) On a level part of the track ($h = 0$) the ball keeps the same speed. Released from rest, the ball reaches $7.0\ \mathrm{m\,s^{-1}}$ after falling $2.5\ \mathrm{m}$. Find k_1 and k_2. [3 marks]

Answer .....

Total for C6: 9 marks

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D1

> Alex writes: "Dimensional analysis shows that $k = 1$, so the speed is exactly $\sqrt{g\lambda}$."

On deep water, the speed v of a wave is modelled as $v = kg^{\alpha}\lambda^{\beta}\rho^{\gamma}$, where λ is the wavelength, ρ is the density of the water and k is a dimensionless constant.

(a) Use dimensional analysis to find α, β and γ. [4 marks]

Answer .....

(b) Explain what $\gamma = 0$ tells you about waves on a liquid that is denser than water. [2 marks]

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(c) Explain why Alex is wrong. [2 marks]

(d) A wave of wavelength $10\ \mathrm{m}$ travels at $3.95\ \mathrm{m\,s^{-1}}$. Find k, to 3 significant figures. [2 marks]

Answer .....

(e) A wave on the same water travels at $6.0\ \mathrm{m\,s^{-1}}$. Find its wavelength. [2 marks]

Answer .....

Total for D1: 12 marks

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D2 A car moves on a level road against air resistance of magnitude cv^2, where v is its speed and $[c] = \mathrm{ML}^{-1}$. Its greatest speed is modelled as $v = kP^{\alpha}c^{\beta}$, where P is the greatest power of the engine and k is a dimensionless constant. Use dimensional analysis to find v in terms of P, c and k, checking that all three index equations are satisfied. A car with $P = 57.6\ \mathrm{kW}$ and $c = 0.9\ \mathrm{kg\,m^{-1}}$ has greatest speed $40\ \mathrm{m\,s^{-1}}$. Find k. The engine is replaced by one with twice the power. Find the percentage increase in the greatest speed. [8 marks]

Answer .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Short ripples on a liquid are driven by surface tension σ, where $[\sigma] = \mathrm{MT}^{-2}$. Their speed is modelled as $v = kg^{\alpha}\lambda^{\beta}\rho^{\gamma}\sigma^{\delta}$, where λ is the wavelength, ρ the density of the liquid and k a dimensionless constant. For very short ripples, the speed does not depend on g. Find the formula for the speed of very short ripples. Hence find the factor by which their speed changes when the wavelength is made 4 times as long. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 There are four unknown powers and three index equations, so write α, β and γ in terms of δ.

Hint 2 "Does not depend on g" tells you the power of g.

Hint 3 Once you have the formula, the speed is proportional to a power of λ: put in the factor 4.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can set up a formula with unknown powers and a constant k, and solve the index equations for M, L and T.	○ ○ ○
I can work out how a quantity changes when another is scaled, and say what a power of 0 means.	○ ○ ○
I can explain why M, L and T cannot fix four unknown powers, and use extra information to finish.	○ ○ ○
I can find a dimensionless constant from data, and explain why dimensional analysis cannot find it.	○ ○ ○

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